A Novel Robust Adaptive Trajectory Tracking in Robot Manipulators

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Abstract

In this paper, a novel adaptive sliding mode control for rigid robot manipulators is proposed. In the proposed system, since there may exist explicit unknown parameters and perturbations, a Lyapunov based approach is presented to increase system robustness, even in presence of arbitrarily large (but not infinite) discontinuous perturbations. To control and track the robot, a continuous controller is designed with two phases of adaptation. The first phase is related to the robot parameters and the other one is accounted for perturbation estimating. We investigated the stability in the sense of Lyapunov with derive adaptive laws and uniform ultimate boundedness in the applied worst condition. The simulation results for two degrees of freedom rigid robot manipulator effectively demonstrate capability of the mentioned approach. Moreover, the results show that the domain of attraction is so vast and a global uniform ultimate boundedness could be expected.

Keywords: *Adaptive Control, Sliding Mode, Perturbation Estimation, Trajectory Tracking, Rigid Robot Manipulators.*

1. Introduction

Sliding mode control (SMC) is one of the most important and simple robust approaches to track control target in presence of large uncertainties, nonlinearities and bounded external disturbances. Therefore, SMC has been widely used in many applications like motion control, robotics, instrumentation and so forth. In some SMCs, although a fixed upper bound is assumed for uncertainties [1], but due to complexity of the structure of uncertainties in robotic manipulators model, achievement to these bounds is not simply possible. Therefore, adaptive methods are proposed to estimate these upper bounds [2, 3, 4]. There are two important phenomena, often undesired, in SMCs. The first one is chattering that will be happened in both conventional SMC (linear SMC) and also nonlinear sliding surface which is called terminal sliding mode control (TSMC). The second one is singularity in TSMC. Terminal sliding mode offers some advantages in comparison with Linear SMC, such as fast and finite time convergence. Nonsingular terminal sliding mode manifolds (NTSMC) are proposed to overcome the second phenomena [5].

To obtain continuous control form, in some researched a virtual discontinuous control input is defined, so that the real control input is obtained just by virtual control integrating. Virtual state and input assumption may cause more complexity in design process [6]. In order to eliminate chattering, the following methods are often used:

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(a) Boundary layer to solve discontinuity in control signal. Some smooth functions like saturation function or tangent hyperbolic must be used instead of sign function. This method may cause more steady state tracking error [7]; (b) Higher order SMC (second order SMC [8, 9], full order SMC [10] and high order ones [11], leads to chattering reduction but may increases control complexity) and (c) Perturbation estimation [12].

Some main features of the proposed adaptive robust controller are categorized as follows: (i) the structured or parametric uncertainties and unstructured uncertainties including unmodeled dynamics and unknown external disturbances are synthesized in a compact term called perturbation and trying to estimate this term is essential. In some usual SMCs, the design process is conservatively based on applying uncertainties upper bounds [13], [14], but with utilizing perturbation estimation method, this conservatism is reduced. (ii) The developed smooth control law eliminates the chattering phenomena without trade-off between performance and robustness, which is prevalent in boundary layer approach. In fact, we try to prevent presence of discontinuity from the first step and successfully provide continuous control effort without any special or common complexities that arised in previous part for solving discontinuity problem [15], [16]. Consequently, there is no need to make trade-off between approximation accuracy and chattering reduction, which is always seen in approximation methods. (iii) In previous works, it is usually assumed that information about some nominal parameters or the uncertainties bounds is known which is used directly in the controller design. In the proposed method, no information about nominal values or bounds is accessible. In fact, completely unknown parameters assumption is more stricter than uncertainty conditions. (iv) Discontinuous and large magnitude terms, such as hard nonlinearities, could be inserted in the perturbation term while preserve satisfactory tracking performance. In this condition, the uniform ultimate boundedness (UUB) is defined in the worst situation. (v) In the presence of time invariant perturbation with arbitrarily large magnitude (but not infinite), such as step form of external disturbance signal, precise tracking will be obtained.

This article is structured as follows. Section 2 is on the system dynamics and problem definition. Section 3 describes the controller design. Robustness and stability proof of the proposed controller is organized in section 4. Section 5 presents the simulation results conducted on two degrees of freedom (DOF) serial rigid robot manipulator. The domain of attraction is examined in the sixth section without proof. To conclude, section 7 is prepared.

2. Problem Definition

Consider an n-link rigid serial robot manipulator as the same as [17]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + f(\dot{q}) = \tau \qquad (1)$$

Where q is the $n \times 1$ vector of joints displacements, $\dot{q}$is the $n \times 1$ vector of joints velocities, $\tau$is the $n \times 1$ vector of input torques, $M(q)$is set as the $n \times n$symmetric positive definite manipulator inertia matrix, $C(q,\dot{q})$is the $n \times n$matrix of centrifugal and Coriolis and $g(q)$ stands for the$n \times 1$ vector of gravitational torques and the $n \times 1$ vector $f(\dot{q})$presents the coulomb (F_c) and viscous (F_v) friction torques [18].

$$M(q,\theta)\ddot{q} + C(q,\dot{q},\theta)\dot{q} + F(\dot{q},\theta) + g(q,\theta) = Y(q,\dot{q},\ddot{q})\theta \qquad (2)$$

Where $Y(.) \in R^{n\times p}$ is the regression matrix of manipulator dynamic equation and $\theta = (\theta_1, \dots, \theta_p)^T$ is the vector containing the unknown manipulator parameters. The dynamic error corresponding to (1) is

$$\begin{cases} \dot{e_1} = e_2 \\ \ddot{e} = \dot{e_2} = M^{-1}(q)[\tau + \tau_d - C(q,\dot{q})\dot{q} - g(q) - f(\dot{q})] - \ddot{q_d} \end{cases} \qquad (3)$$

Property 1

The inertia matrix $M(q)$ is bounded, such that $\mu_m I_n \le M(q) \le \mu_M I_n$,$\forall\, q \in R^n$ and some positive constants $\mu_m \le \mu_M$.

Property 2

The matrix $C(q,\dot{q})$ satisfies [4]

$$S^T\left[\dot{M}(q) - 2C(q,\dot{q})\right]S = 0 \;\;,\;\; \forall\; S \in R^n$$

Property 3

If $|.|$ stands for absolute (.), then

$$A^T_{(1\times n)}B_{(n\times p)}C_{(p\times 1)} \le |A|^T|B||C|$$

$$A_{(n\times 1)} = diag\big(sign(A)\big)|A|_{(n\times 1)}$$

3. Controller Design

The tracking control problem can be achieved by keeping the system trajectory on the sliding surface equal to zero. This surface is defined as [19].

$$S(t) = \dot{e} + \Lambda e + \beta \int e \, dt \quad (4)$$

$$S = \dot{q} - \dot{q}_r \quad (5)$$

Where $S(t)$is $n \times 1$ vector, Λ is $n \times n$diagonal and positive definite constant matrix and $e = q - q_d, \dot{e} = \dot{q} - \dot{q}_d$are the position error and the position rate error.

The compact unknown uncertainties which is called perturbation term $\tilde{F}$ could be set as

$$\tilde{F} = \tau_d - F_c \quad (6)$$

To test the level of control robustness, the coulomb friction could be assumed as robot parameters uncertainty and applied without compensating.

Assumption 1

The unknown perturbation term $\tilde{F}$ is not infinite but could be discontinuous

$$\|\tilde{F}\| < \infty$$

Assumption 2

An upper bound is defined for the rate of perturbation which may increases up to infinity.

$$\|\dot{\tilde{F}}\| \leq a$$

These bounds are not required to be known or estimated, and never used in control and adaptive laws design. These bounds may just appear in ultimate boundedness proof procedure.

The next step of control design is to choose a control law such that it leads the Lyapunov candidate be a decreasing function over time. This following control law is set as

$$u(t) = u_0 + u_p \quad (7)$$

Where u_0 is considered to control system when robot parameters are completely unknown and in the absence of perturbations

$$u_0 = Y\left(q, \dot{q}, \ddot{q}_r\right)\hat{\theta} - K_P S - K_D \dot{S} \quad (8)$$

The $K_P S$ presents a PID controller to enhance stability of closed-loop system and will improve the transient performance, $K_D \dot{S}$ is considered to make the sliding surface smoother and may causes faster convergence.

The term u_p is designed to compensate the perturbation effect and estimate it.

$$u_p = -\tilde{F}_{est} \quad (9)$$

Finally, the following control law is proposed:

$$u(t) = Y(q, \dot{q}, \ddot{q}_r)\hat{\theta} - K_P S - K_D \dot{S} - \tilde{F}_{est} \quad (10)$$

The system parameters and perturbation adaptation laws are derived separately as

$$\begin{cases} \dot{\hat{\theta}} = -\frac{\gamma}{\lambda_{min}} Y^T sign(Y \tilde{\theta}) sign(S)\, S = -\frac{\gamma}{\lambda_{min}} Y^T B\ |S| \\ \dot{\tilde{F}}_{est} = -\frac{\Gamma}{\lambda_{min}} sign(E) sign(S)\, S = -\frac{\Gamma}{\lambda_{min}} A\ |S| \end{cases} \quad (11)$$

Where B=$sign\left(Y\tilde{\theta}\right)$, $A = sign(E)$ and E is the vector of the lumped uncertainty estimation error, such that

$$E = \tilde{F}_{est} - \tilde{F} \quad (12)$$

The vector θ is time-invariant and implies the system real parameters, $\hat{\theta}$ is parameters estimation vector and $\tilde{\theta} = \hat{\theta} - \theta$ is the error of system parameters estimation, that yields

$$\begin{cases} \hat{\theta} = -\frac{\gamma}{\lambda_{min}} \int Y^T B\, |S| \\ \tilde{F}_{est} = -\frac{\Gamma}{\lambda_{min}} \int A\ |S| \end{cases} \quad (13)$$

4. Stability Analysis

To prove the robustness and stability of the proposed controller the following Lyapunov function is defined

$$V = \frac{1}{2}\left(S^T S + E^T \Gamma^{-1} E + \tilde{\theta}^T \gamma^{-1} \tilde{\theta}\right) + a\, b \int e^T e \quad (14)$$

Where Γ and γ are diagonal positive definite constant matrices. The parameters a and b are positive constant scalars.

In this analysis $|.| = abs(.)$ as said previously and $K_D$is PD, so according to property 1,

$$\mu_m I + K_D \leq M + K_D \leq \mu_M I + K_D$$

$$\equiv\ Z \leq \alpha \leq \xi$$

Since Z , α , ξ , M are PDs, therefore

$$\xi^{-1} \leq \alpha^{-1} \leq Z^{-1} \quad (15)$$

The derivative of Lyapunov function is

$$\dot{V} = S^T\dot{S} + E^T\,\Gamma^{-1}\dot{E} + \tilde{\theta}^T\,\gamma^{-1}\dot{\tilde{\theta}} + a\,b\,(e^T e) \quad (16)$$

Bysubstituting $\dot{S} = \alpha^{-1}Y\tilde{\theta} + \alpha^{-1}(\tau_d - F_c) - \alpha^{-1}\tilde{F}_{est} - \alpha^{-1}K_P S$ and applying property 3, we have

$$\dot{V} \leq |S|^T\alpha^{-1}\left|Y\,\tilde{\theta}\right| + |S|^T\alpha^{-1}|E| - S^T\alpha^{-1}K_P\,S + E^T\,\Gamma^{-1}\dot{\tilde{F}}_{est} - E^T\,\Gamma^{-1}\dot{\tilde{F}} + \tilde{\theta}^T\,\gamma^{-1}\dot{\tilde{\theta}} + a\,b\,|e|^T|e|$$

And

$$E = diag\big(sign(E)\big)|E| = A|E| \Rightarrow |E| = A^{-1}\,E$$

$$Y\,\tilde{\theta} = diag\left(sign(Y\,\tilde{\theta})\right)\left|Y\,\tilde{\theta}\right| = B\left|Y\,\tilde{\theta}\right|$$

$$\Rightarrow \left|Y\,\tilde{\theta}\right| = B^{-1}Y\,\tilde{\theta}$$

$$S = diag\big(sign(S)\big)|S| = C\,|S| \quad \Rightarrow \quad |S| = C^{-1}S$$

According to (15), following nonequality is used instead of (16)

$$\dot{V} \leq |S|^T Z^{-1}\left|Y\,\tilde{\theta}\right| + |S|^T Z^{-1}|E| - S^T\alpha^{-1}K_P\,S + E^T\,\Gamma^{-1}\dot{\tilde{F}}_{est} - E^T\,\Gamma^{-1}\dot{\tilde{F}} + \tilde{\theta}^T\,\gamma^{-1}\dot{\tilde{\theta}} + a\,b\,|e|^T|e|$$

Because Z is PD, the following nonequalities are established:

$$\lambda_{min}I \leq Z \leq \lambda_{max}I \quad \xrightarrow{yields} \frac{1}{\lambda_{max}}\,I \leq Z^{-1} \leq \frac{1}{\lambda_{min}}\,I,$$

$$\dot{V} \leq \frac{1}{\lambda_{min}}\tilde{\theta}^T\,Y^T B^{-1}\,C^{-1}\,S + \frac{1}{\lambda_{min}}E^T\,A^{-1}\,C^{-1}\,S - S^T\alpha^{-1}K_P\,S + E^T\,\Gamma^{-1}\dot{\tilde{F}}_{est} - E^T\,\Gamma^{-1}\dot{\tilde{F}} + \tilde{\theta}^T\,\gamma^{-1}\dot{\tilde{\theta}} + a\,b\,\|e\|^2$$

Where$\|.\|$ denotes the norm of function (.). Eventually, the adaptive laws are derived as (11) however some terms are remaining yet as follows:

$$\dot{V} \leq -S^T\alpha^{-1}K_P\,S - E^T\,\Gamma^{-1}\dot{\tilde{F}} + a\,b\,\|e\|^2 \quad (17)$$

In this section, following three different cases will appear:

Case 1

If rate of perturbation be so small such that it could be ignored ($\dot{\tilde{F}} \cong 0$) or when perturbation is time invariant, the upper bound of perturbation rate (a) will be assumed zero and it means that the second and third right hand terms of (17) will be omitted. The reduced form of (17) is

$$\dot{V} \leq -S^T\alpha^{-1}K_P\,S$$

Which satisfies the system closed loop stability because $\alpha^{-1}K_P$ is PD.

Case 2

If error of perturbation estimation be equal to zero ($E \cong 0$) and $\tilde{F}$ be time varying too, uniform ultimate boundedness should be defined for system.

$$-S^T\alpha^{-1}K_P\,S - E^T\,\Gamma^{-1}\dot{\tilde{F}} + a\,b\,\|e\|^2 \leq 0,$$

$$a\,b\,\|e\|^2 \leq S^T\alpha^{-1}K_P\,S \leq \|S\|^2\|\alpha^{-1}K_P\|$$

Property 4

After reaching phase, an upper bound could be defined for sliding manifold as follows:

$$\|S\| \leq \varepsilon$$

Where this bound usage is just in ultimate boundedness proof procedure.

The last property and using the equality $\|D\| = \sqrt{\lambda_{max}(D)}$for $n \times n$ matrix D, such that $\lambda_{max}(D)$ is set as maximum of eigenvalues of D, lead to

$$a\,b\,\|e\|^2 \leq \varepsilon^2\sqrt{\lambda_{max}(\alpha^{-1}K_P)} \leq \varepsilon^2\sqrt{\lambda_{max}(Z^{-1}K_P)} \leq \varepsilon^2\,\varrho \xrightarrow{yields} \|e\| \leq \varepsilon\sqrt{\frac{\varrho}{a\,b}} \quad (18)$$

Case 3

If both E and $\dot{\tilde{F}}$ are nonzero (the most complex condition),

$$\dot{V} \leq -E^T\,\Gamma^{-1}\dot{\tilde{F}} + a\,b\,\|e\|^2 \leq 0$$

From assumption 2 and by substituting $\sqrt{\lambda_{max}(\Gamma^{-1})} = \vartheta$ and $\|E\| = \epsilon$, the uniform ultimate boundedness in this case is as follows

$$\|e\| \leq \sqrt{\frac{\|E\|\big(\lambda_{max}(\Gamma^{-1})\big)^{1/2}}{b}} \xrightarrow{yields} \|e\| \leq \sqrt{\frac{\epsilon\,\vartheta}{b}} \quad (19)$$

It satisfies stability even in the worst condition which the rate of perturbation is infinity. So this controller will handle the discontinuous perturbation terms like coulomb friction. Although at the times of discontinuities occurrence, it is impossible to have zero perturbation estimation error, because perturbation estimation term has always continuous form, as mentioned previously in (12).

Assumption 3

The upper bounds of $\|E\|$ and $\|S\|$ can be obtained visually from their plots in simulation results.

Assumption 4

The end effector is assumed as a point which is placed at the end part of final link.

5. Simulation Results

To illustrate control performance and robustness of presented controller, simulation results for a two DOF serial robot is investigated which the dynamic model offered here [17].

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + f(\dot{q}) = \tau$$

Where

$$M(q) = \begin{bmatrix} \theta_1 + 2\theta_2\cos(q_2) & \theta_3 + \theta_2\cos(q_2) \\ \theta_3 + \theta_2\cos(q_2) & \theta_3 \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} -2\theta_2 \, sin(q_2)\,\dot{q}_2 & -\theta_2 \, sin(q_2)\,\dot{q}_2 \\ \theta_2 \, sin(q_2)\,\dot{q}_1 & 0 \end{bmatrix}$$

$$g(q) = \begin{bmatrix} \theta_4 \sin(q_1) + \theta_5\sin(q_1 + q_2) \\ \theta_5\sin(q_1 + q_2) \end{bmatrix}$$

$$f(\dot{q}) = \begin{bmatrix} \theta_6\dot{q}_1 + \theta_8 sgn(\dot{q}_1) \\ \theta_7\dot{q}_2 + \theta_9 sgn(\dot{q}_2) \end{bmatrix}$$

Advantages of this type of direct-drive actuator include freedom from backlash [20]. The desired reference signals are given in [18].

$$q_d(t) = \begin{pmatrix} q_{d1}(t) \\ q_{d2}(t) \end{pmatrix} = \begin{pmatrix} \frac{\pi}{2} + \sin(\omega t) \\ cos(\omega t) \end{pmatrix} [\text{rad}] \quad (20)$$

The controller design parameters are set as

$$\Lambda = \begin{bmatrix} 3 & 0 \\ 0 & 8 \end{bmatrix} , \quad \beta = \begin{bmatrix} 4 & 0 \\ 0 & 10 \end{bmatrix}$$

$$K_P = \begin{bmatrix} 400 & 0 \\ 0 & 200 \end{bmatrix} , \quad K_D = \begin{bmatrix} 150 & 0 \\ 0 & 70 \end{bmatrix} ,$$

$$\Gamma = \begin{bmatrix} 15000 & 0 \\ 0 & 10000 \end{bmatrix} , \quad \gamma = diag([2 \;\; 2 \;\; 5 \;\; 2 \;\; 4 \;\; 10 \;\; 3])$$

$$\theta = [2.351 \;\; 0.084 \;\; 0.102 \;\; 2.288 \;\; 0.175 \;\; 38.465 \;\; 1.825]^T$$

The initial conditions is supposed as follows

$$\theta(0) = [\, 2.88 \;\; 0.103 \;\; 0.125 \;\; 2.803 \;\; 0.214 \;\; 47.119 \;\; 2.235 \,]^T$$

$$\begin{bmatrix} q(0) \\ \dot{q}(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The compared method [12], design a control while the perturbation term is explicitly unknown but there is uncertainty in robot parameters. In fact the parameters are uncertain not completely unknown. Hence, the compared methods should be in the same situation as the main method, we tried to apply some appropriate changes in this compared method. The old and new features of this compared approach are demonstrated in Table 1. The external disturbance and coulomb friction amplitudes are more than 10 times and more than %100 of the nominal input torques amplitude respectively, as follows

$$\begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} = \begin{bmatrix} 200cos\big(1.7\pi(t+12)\big) + 300\, sin(\pi t) \\ 100\, sin\big(1.35\pi(t+12)\big) + 50\, cos(2t) \end{bmatrix} \quad (21)$$

$$F = \begin{bmatrix} F_{c1} \;\; sign(\dot{q}_1) \\ F_{c2} \;\; sign(\dot{q}_2) \end{bmatrix} = \begin{bmatrix} 50\; sign(\dot{q}_1) \\ 30\; sign(\dot{q}_2) \end{bmatrix}$$

As can be seen, some of these figures are gathered to compare three cases. (a) Proposed method with coulomb friction compensating,(b)Proposed method without coulomb friction compensating, and (c) Compared method in the new mode, which is mentioned in Table 1.

Figure 1 depicts the nominal applied torque to each joint and phase portrait in presence of step form of external disturbance. Indeed, this figure represents that the mentioned method could handle the control purpose even in presence of arbitrarily large time-invariant perturbation. As it is clear, the magnitude of applied step signal, as an external disturbance, is more than 10 times of nominal torques amplitude. In this situation, precise tracking is happened.

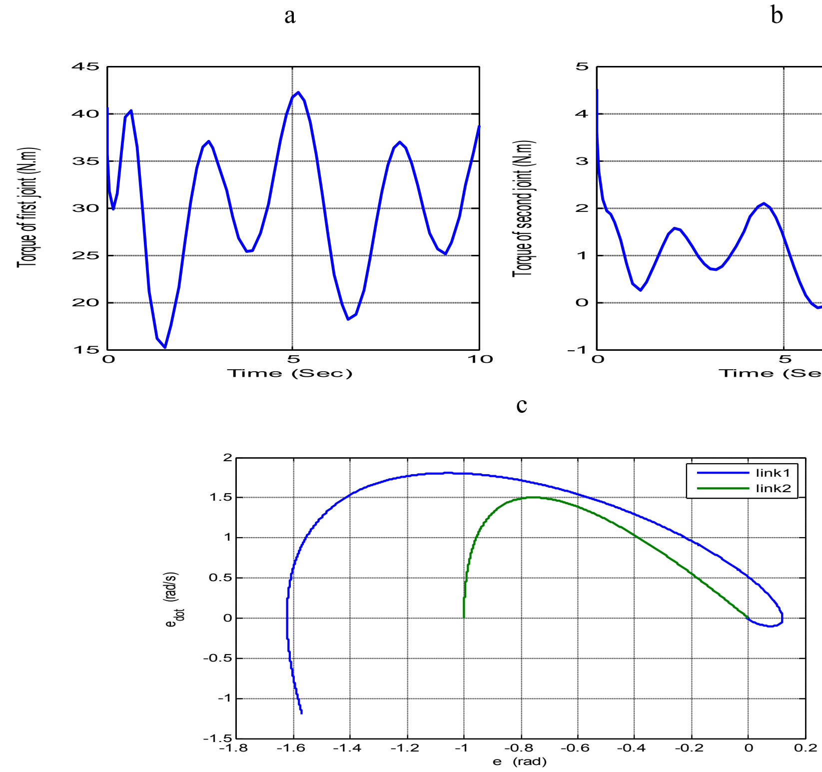

Fig.1. (a) Nominal torque of first joint, (b) Nominal torque of second joint, (c) Phase portrait in presence of large magnitude step disturbance $([d_1(t) \quad d_2(t)] = [400 \quad 200](Nm))$

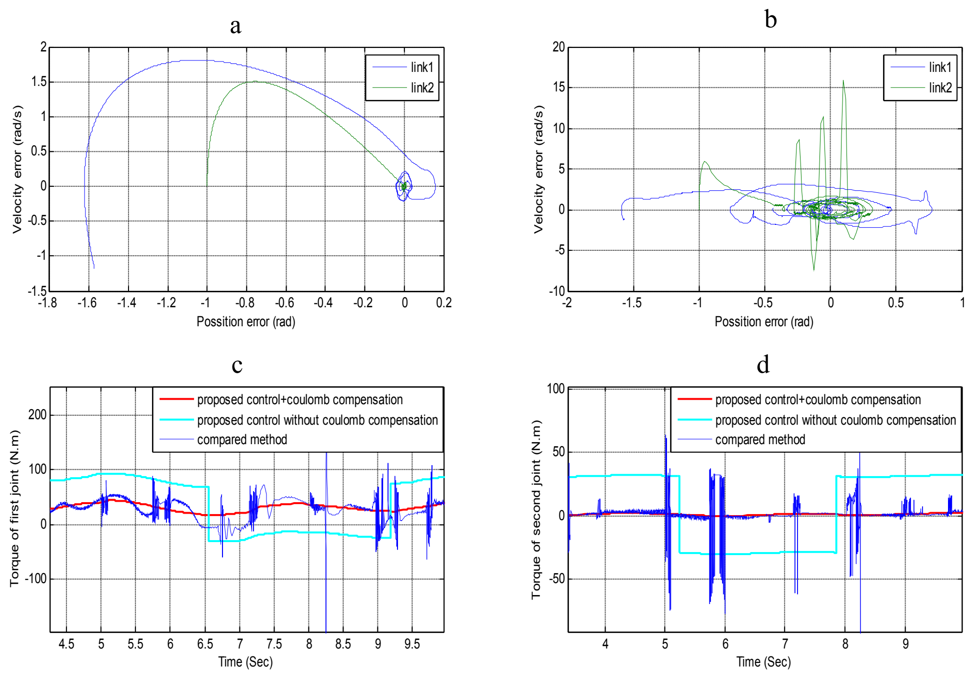

Fig. 2. (a) Phase plane of proposed controller, (b) Phase plane of compared method, (c) Zoomed applied torque to first joint for three cases, (d) Zoomed applied torque to second joint for three cases

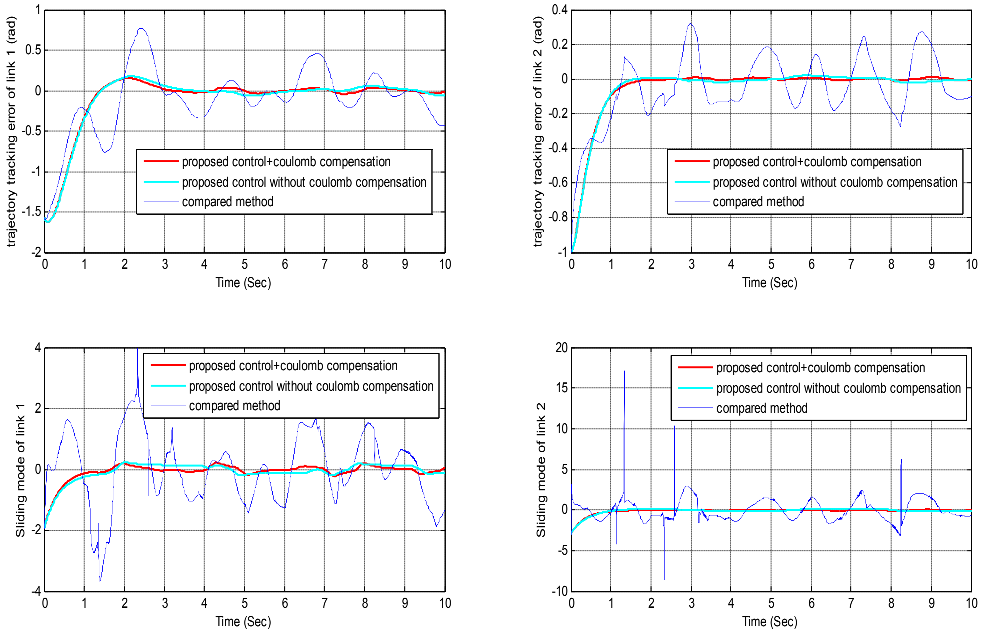

Fig. 3. Trajectory tracking and sliding surface for first and second link (and end effector) for three cases

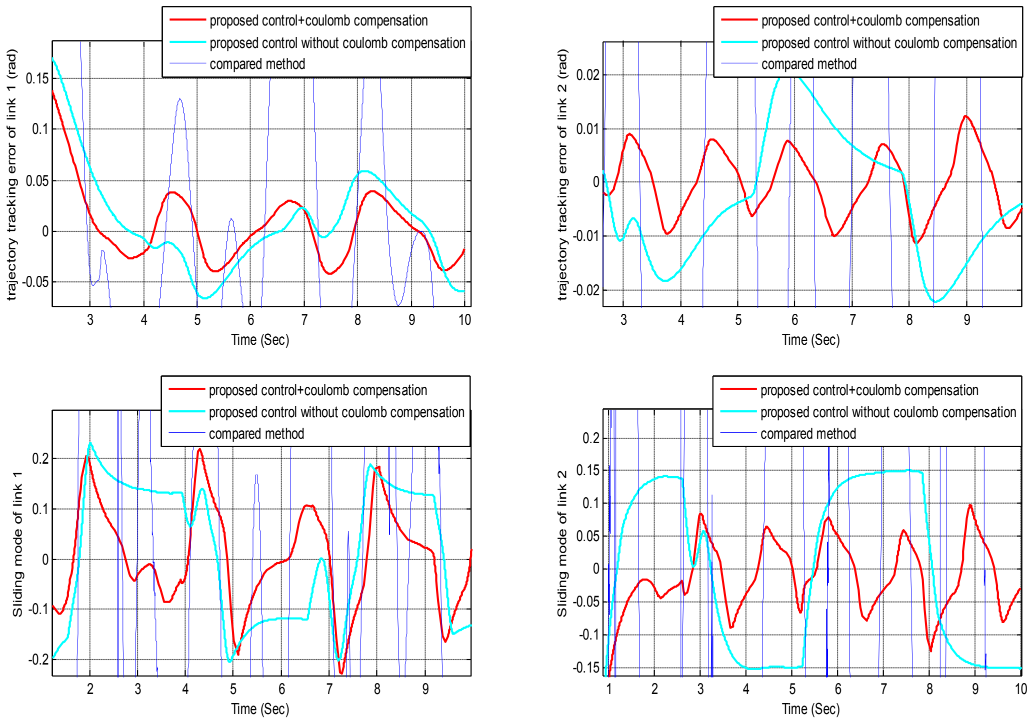

Fig. 4. Zoomed trajectory tracking and sliding surface for first and second link (and end effector) for three cases

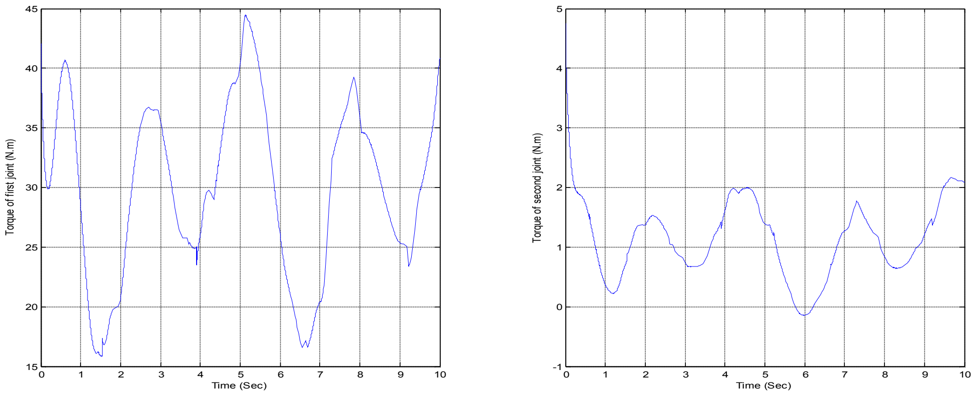

Fig.5. Applied torques to the first and second joint in the worst condition

Table.1. Definition of the compared method functions

	Old (parameters uncertainties)[12]	New (explicitly unknown parameters)
Manipulator dynamic equation	$\tau = M(q,t)\ddot{q} + h(q,\dot{q},t)$	$\tau \equiv Y\theta$
Nominal and uncertainties function	$h(q,\dot{q},t) = \hat{h}(q,\dot{q},t) + \Delta h(q,\dot{q},t)$ $M(q,t) = \widehat{M}(q,t) + \Delta M(q,t)$	...
Sliding surface	$S = \dot{e} + 2\Lambda\, e + \Lambda^2 \int e\, dt = \dot{q} - \dot{q}_r$	$S = \dot{e} + \Lambda\, e + \beta \int e\, dt = \dot{q} - \dot{q}_r$
Lyapunov function	$V = \frac{1}{2}\left(S^T S + E^T \Gamma^{-1} E\right)$	$V = \frac{1}{2}\left(S^T M\, S + E^T \Gamma^{-1} E + \tilde{\theta}^T \gamma^{-1} \tilde{\theta}\right)$
Control input	$u(t) = \widehat{M}\,\ddot{q}_r + \hat{h} - \widehat{M}\,K\,S - \widehat{M}\;\tilde{F}_{est}$	$u(t) = Y(q,\dot{q},\ddot{q}_r)\,\hat{\theta} - K_P\,S - \tilde{F}_{est}$
Adaptive laws	$\dot{\tilde{F}}_{est} = \Gamma\, S$	$\dot{\hat{\theta}} = -\gamma\, Y\, S$ $\dot{\tilde{F}}_{est} = \Gamma\, S$

Figure 2 illustrates the phase plane analysis of proposed and compared methods, and the control input signals (in the worst condition). In figure 3, the position errors and sliding signals are demonstrated. Figure 4 is just a zoomed view of figure 3 in order to compare better the novel approach with coulomb compensating and without compensating.

In the figures which displays all three cases together, the red, green and blue lines are related to the proposed method with coulomb compensating, the proposed control without coulomb compensating and the compared method, respectively.

As we can see in figure 4, for the proposed control (with coulomb compensating), uniform ultimate boundedness (UUB) for the first and second link, is approximately obtained as follows:

$$\begin{cases} \|e_1\| \le 0.06 \quad [rad] \\ \|e_2\| \le 0.03 \quad [rad] \end{cases}$$

Which is satisfactory in the presence of unknown large and discontinuous perturbation.

The motors used in the robot manipulator are the DM1200-A and DM1015-B models from Parker Compumotor for the shoulder and elbow joints (the first and second joints), respectively. In the mentioned configuration, the motor DM1200-A is capable to deliver 200 Nm torque output and the motor DM1015-B only 15 Nm [17]. From figure 5, it is clear that in the worst condition, which is introduced previously, torques of joints always remain in operating bounds even in initial conditions. This feature is realistic and ideal.

6. Domain of Attraction

As said previously, all the results were achieved in the initial condition of $[q_1(0) q_2(0)] = [0 \quad 0]$. According to (20), the initial desired positions below are evident:

$$q_d(0) = \begin{bmatrix} 1.57 \\ 1 \end{bmatrix} \quad (rad)$$

Hence the maximum absolute error in rotational space is $\pi(rad)$, this upper bound should be applied to examine globalization. To check the domain of attraction, the following table will be useful. The new absolute initial trajectory errors are chosen almost as big as the possible maximum error.

Table. 2. Initial positions and errors

Parameters	Previous values (rad)	New values (rad)
$q_1(0)$	0	4
$q_2(0)$	0	5
$\|e_1(0)\|$	1.57	2.43
$\|e_2(0)\|$	1	2.28
$\max\|e\|$	3.14	3.14

Figures 6 and 7 display the powerful tracking in joint space and planar movement of end effector in X-Y plane, respectively (in the worst applied condition with new initial positions which are mentioned in Table 2). From these figures, this fact is inferred that the mentioned approach could provide a wide domain of attraction. This feature emphasizes the concept of perfect robustness.

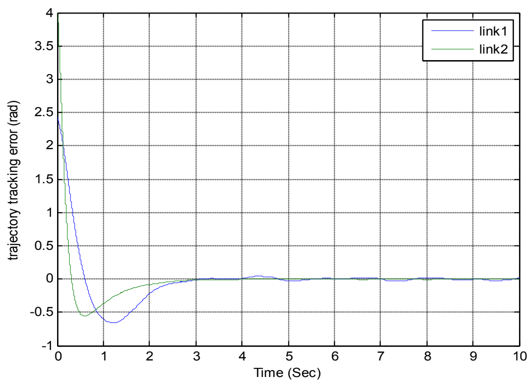

Fig. 6. Trajectory tracking error for new initial positions

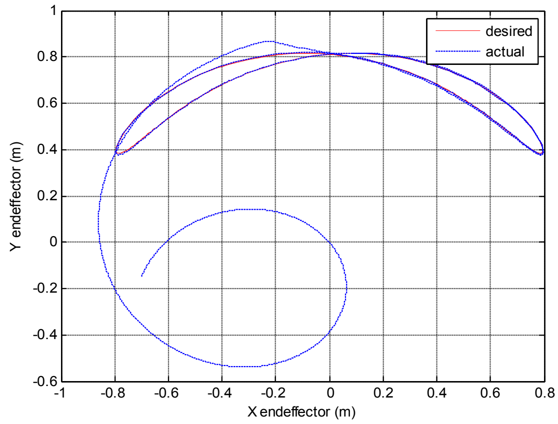

Fig. 7. End effector trajectory tracking error in X-Y plane

7. Conclusion

In this paper, a novel smooth robust adaptive sliding mode control for robot manipulators is proposed. The trajectory tracking aim is achieved with perfect robustness stability in the special condition that neither nominal values nor the bounds of perturbation are available. All unknown disturbances and high frequency unmodeled dynamics or hard nonlinearities are gathered in a lumped term which named perturbation function. Due to the existing practical bound constraints on joint actuators, the proposed method could be implemented on experimental manipulators according to the achievement of smooth and realistic control input. In general, each trade-offs and conservatisms in problem definition and design process may increase the system sensitivity and decrease the control performance.

In this paper, we tried to relax these sensitivities in order to acquire both perfect robust performance and proper robust stability. The simplicity in design is one of the significant features of the offered controller. The simulation results represent that the proposed control when large coulomb friction inserted directly in robot dynamic equation, without compensation, leads to discontinuities in control input. It is clear that in this case, the robustness of manipulator is perfect and also the tracking purpose is handled well. By considering this hard nonlinearity in the perturbation term and trying to compensate it during the estimation process, the control input transforms into a continuous signal and the tracking performance will be improved. The compared method acts inappropriately in the same situation and is weak because of necessity to make a trade-off between the control input torque bound and the ultimate boundedness.

Due to the proof procedure and the simulation results, the precise tracking is accessible in presence of arbitrarily large time-invariant perturbations. The perfect domain of attraction can be a consequence of high robustness. Although in this paper this claim is not proved, it is reasonable based on the results. In other words, the global uniform ultimate boundedness (GUUB), which is ideal, may be confirmed due to the proof process.

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