A New Approach to the Multi-Objective Control of Linear Singular Perturbation Systems

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Abstract

The purpose of this study was to design a logic-based switching H_2/H_∞ state-feedback controller for continuous-time LTI singular perturbation systems. To this end, a hybrid control scheme based on a Genetic Algorithm (GA)-based supervisor is proposed which manages the combination of two controllers. A convex LMI-Based formulation of both fast and slow subsystem controllers leads to a structure which ensures a good performance in both transient and steady state phases. The stability analysis uses Lyapunov techniques, inspired from switching system theory, to prove that a system with the proposed controller remains globally stable despite the configuration (controller) changing.

Keywords: Continuous-time LTI singular perturbation systems, GA-based supervisor, switching H_2/H_∞ state-feedback control, Linear Matrix Inequality (LMI).

1. Introduction

Singular perturbation systems are studied extensively in numerous papers and books (see for example [3],[4],[8],[12],[13]). Many approaches such as optimal and robust control schemes have been considered for these systems of both linear and nonlinear types. For the robust control of singular perturbation systems, the controller is usually derived from indirect mathematical programming approaches (e.g. solving Riccati equations), which encounters serious numerical problems because of the stiffness of the equations involved in the design. To avoid this difficulty, several approaches [6], [9] have been developed to transform the original problem into ϵ-independent sub-problems, among which the time-scale decomposition [6] is commonly adopted. As an alternative to the solution of the Riccati equation, LMI formulations have been attracting more and more attention of robust control researchers. However, solving mixed H_2/H_∞ perturbation systems through the LMI approach still remains an open question. Among those researchers who worked on LMI formulations, Garcia et al. [4] extended the results of a study by Peres and Gromel [14] and suggested a solution to the infinite time near optimal regulator problem (H_2 control) for singular perturbation systems through an LMI formulation. A time-scale decomposition was employed on the overall system as well. Li et al. [15] devised a different way for solving this problem. By proposing a new lemma, they formulated the problem into a set of inequalities independent of ϵ., and provided an algorithm to solve them through LMI formulations. In [16] the H_2 static state feedback control of linear singular perturbation systems by extending [15] was presented, but the further extension of this method to the mixed H_2/H_∞ control is very difficult. Combining different techniques to obtain different performances is widely used today ([7], [10], and [11]). It results in hybrid dynamical systems which include continuous and discrete dynamics and a

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mechanism (supervisor) managing the interaction between the dynamics [2].

Compared with the sole H_∞ control, the mixed H_2/H_∞ control is more attractive to engineering practices since the former is a worst-case design which tends to be conservative whereas the later minimizes the average performance with a guaranteed worse-case performance. In [17, 18] a new approach based on the logic-based switching control [19] was considered for the multi-objective mixed H_2/H_∞ control of linear singular perturbation systems. In these two papers, by utilizing the results of [1, 6, 9] on the multi-objective control approach and designing a fuzzy supervisor based on [7, 10], a control signal was constructed with the weighted sum of two control signals for both slow and fast subsystems while the weighted factor was governed online by the fuzzy supervisor. In [20] an extension of the methodology used in [16] was for the H_∞ control of singular perturbation systems.

The present study also deals with continuous-time linear singular perturbation systems. Yet its main novelty lies in the fact that the switching mixed H_2/H_∞ state feedback control problems of continuous-time linear singular perturbation systems are solved by a different design structure for the supervisor. The simple design methods of [1] are applied to derive the state-feedback gains separately for fast and slow sub-systems. A GA-based supervisor (instead of a fuzzy supervisor) which is proposed for the hybrid combination of the controllers and their corresponding advantages is able to ensure the required performances and the stability of the overall closed loop system. The contribution of the present work is combining fast and slow sub-system controllers using a GA-based supervisor which manages the gradual transition from one controller to another. The control signal is obtained via the weighted sum of the two signals provided by the slow and fast sub-system controllers. This weighted sum is managed thanks to a GA-based supervisor, which is adapted to obtain the desired closed loop system performances.

As a result, while the fast sub-system controller mainly acts in the transient phase providing a fast dynamic response and enlarging the stability limits of the system, the slow sub-system controller mostly works in the steady state to reduce chattering and maintain the tracking performance. Furthermore, the global stability of the system, even in the case that the system switches from one configuration to another (from the transient to the steady state and vice versa) is guaranteed.

The structure of the paper is as follows. Section 2 presents the definition of systems and the controllers used. In Section 3, the GA-based supervisor and the proposed control law are described, and the stability analysis is demonstrated in Section 4. The design procedure is explained in Section 5 and an example is given to illustrate the efficiency of the proposed method, followed by the conclusions in Section 6.

2. Statement of Problem

Consider the following linear singularly perturbated system Σ with slow and fast dynamics described in the "singularly perturbated" form [17]:

$$\Sigma:\begin{cases}\dot{x}_1 = A_1x_1 + A_2x_2 + B_{w_1}w + B_1u\\ \epsilon\dot{x}_2 = A_3x_1 + A_4x_2 + B_{w_2}w + B_2u\\ z = C_{z_1}x_1 + C_{z_2}x_2 + D_zu\\ y = C_1x_1 + C_2x_2\end{cases} \tag{1}$$

Where $x_i\epsilon R^{n_i}, i = 1, 2$ are the states, $u\epsilon R^{m_1}$ is the control input, $w\epsilon R^{m_2}$ is the disturbance input, $y\epsilon R^{l_1}$ is the measured output, $z\epsilon R^{l_2}$ is the output to be regulated, and ϵ is a small positive parameter. By introducing this notation:

$$x = \begin{bmatrix}x_1\\x_2\end{bmatrix}, A_\epsilon = \begin{bmatrix}A_1 & A_2\\ \frac{1}{\epsilon}A_3 & \frac{1}{\epsilon}A_4\end{bmatrix}, B_\epsilon = \begin{bmatrix}B_1\\ \frac{1}{\epsilon}B_2\end{bmatrix},\ B_{w_\epsilon} = \begin{bmatrix}B_{w_1}\\ \frac{1}{\epsilon}B_{w_2}\end{bmatrix} \tag{2}$$

$$C_z = [C_{z_1}\quad C_{z_2}],\quad C = [C_1\quad C_2]$$

System Σ can be rewritten into the following compact form:

$$\Sigma:\begin{cases}\dot{x} = A_\varepsilon x + B_{w_\varepsilon}w + B_\varepsilon u\\ z = C_zx + D_zu\\ y = Cx\end{cases} \tag{3}$$

Then applying a static state feedback control

$$u = Kx \tag{4}$$

leads to the following closed-loop system:

$$\Sigma_{cl}:\begin{cases}\dot{x} = A_{cl}x + B_{cl}w\\ z = C_{cl}x\end{cases} \tag{5}$$

where

$$A_{cl} = A_\varepsilon + B_\varepsilon K, B_{cl} = B_{w_\varepsilon}, C_{cl} = C_z + D_zK \tag{6}$$

denote the transfer function of closed-loop system Σ_{cl} from w to z as $T(s, K) = C_{cl}(sI - A_{cl})^{-1}B_{cl}$.

The generalized H_2norm of $T(s, K)$ is defined by:

$$\|T(s,K)\|_2 = \tfrac{1}{2\pi}\int_{-\infty}^{+\infty} tr(T^*(j\omega,K)T(j\omega,K))d\omega \tag{7}$$

and the H_∞ norm of $T(s, K)$ is defined by:

$$\|T(s,K)\|_\infty = \sup_{\|w\|_2\neq 0}\frac{\|z\|_2}{\|w\|_2} \tag{8}$$

2.1. Slow and Fast Sub-systems

If A_4 is a nonsingular matrix, the original singularly perturbated system in (1) can be decomposed into a slow and a fast subsystem. The slow subsystem is defined by setting $\varepsilon = 0$ in the second equation of (1), computing x_2 in terms of x_1,w, u, and finally substituting it in the first equation of (1). Therefore, the slow subsystem obtained is as follows [17]:

$$\begin{cases}\dot{x}_s = A_sx_s + B_{w_s}w + B_{1_s}u_s\\ z_s = C_{z_s}x_s + D_{w_s}w + D_su_s\\ y_s = C_sx_s + D_{w_{s1}}w + D_{s1}u_s\end{cases} \tag{9}$$

where:

$$A_s = A_1 - A_2A_4^{-1}A_3$$
$$C_{z_s} = C_{z_1} - C_{z_2}A_4^{-1}A_3\quad D_{w_s} = -C_{z_2}A_4^{-1}B_{w_2} \tag{10}$$

$\mathrm{B_{w_s} = B_{w_1} - A_2A_4^{-1}B_{w_2}} \quad \mathrm{B_{1_s} = B_1 - A_2A_4^{-1}B_2}$
$\mathrm{D_s = D_z - C_{z_2}A_4^{-1}B_2} \quad \mathrm{C_s = C_1 - C_2A_4^{-1}A_3}$
$\mathrm{D_{w_{s1}} = -C_2A_4^{-1}B_{w_2}} \quad \mathrm{D_{s1} = -C_2A_4^{-1}B_2}$

The fast subsystem of (1) is defined by:

$$\begin{cases} \varepsilon\dot{x}_f = A_4x_f + B_{w_2}w + B_2u_f \\ z_f = C_{z_2}x_f + D_zu_f \\ y_f = C_2x_f \end{cases} \tag{11}$$

Therefore, according to (9) and (11) the overall full order system in (1) can be decomposed into two slow and fast subsystems with lower orders. Then these two subsystems can be used for the slow and fast controller design and can be mixed through the utilization of a GA supervisor to produce the proposed controller for the overall system. It should be noted that this article deals with the suboptimal mixed $\mathrm{H_2/H_\infty}$ static state feedback control design problem. Consequently, the suboptimal H_2, H_∞ and mixed $\mathrm{H_2/H_\infty}$ problems are expressed in terms of linear matrix inequalities (LMI).

● *Lemma 2.1.* [1] (The suboptimal overall H2 static state feedback control problem):

Consider the overall system described by (1). The static state feedback control law (4) stabilizes the closed-loop system (5) and achieves a prescribed H2-norm bound $\upsilon > 0$ for system (5) if and only if there exists $\mathrm{Q = Q^T >} 0, T$ and Z with appropriate dimensions such that:

$$\begin{pmatrix} AQ + QA^T + B_\epsilon T + T^T {B_\epsilon}^T & QC_z^T + T^T {D_z}^T \\ C_zQ + D_zT & -I \end{pmatrix} < 0$$
$$\begin{pmatrix} Q & B_{w_\varepsilon} \\ {B_{w_\varepsilon}}^T & Z \end{pmatrix} > 0 \tag{12}$$
$$\mathrm{Trace(Z)} < \nu$$

By solving the LMIs, Q, T and Z are found, and control law (4) is calculated as:

$$\mathrm{K = TQ^{-1}} \tag{13}$$

The application of (13) to the system characterized by (1) guarantees that the closed-loop system (5) is asymptotically stable and that H2-norm (7) is less than $\upsilon > 0$.

● *Lemma 2.2.* [1] (The suboptimal overall H∞ static state feedback control problem)

Consider the overall system specified by (1). The static state feedback control law numbered (4) stabilizes closed-loop system (5) yielding a prescribed H∞-norm bound $\boldsymbol{\gamma > 0}$ for closed-loop system (5) if and only if there exists $\boldsymbol{Q = Q^T > 0}$ ***and*** $\boldsymbol{T}$ with appropriate dimensions such that:

$$\begin{pmatrix} \mathbf{AQ + QA^T + B_\epsilon T + T^T {B_\epsilon}^T} & \mathbf{B_{w_\varepsilon}} & \mathbf{QC_z^T + T^T {D_z}^T} \\ \mathbf{{B_{w_\varepsilon}}^T} & \mathbf{-I} & \mathbf{D_{cl}^T} \\ \mathbf{C_zQ + D_zT} & \mathbf{D_{cl}} & \mathbf{-\gamma^2 I} \end{pmatrix} \mathbf{< 0} \tag{14}$$

By solving LMI (14), Q and T are found, and control law (4) is calculated from (13).

The application of this controller to the system defined by (1) guarantees that closed-loop system (5) is asymptotically stable and that H∞-norm (8) is less than $\gamma > 0$.

● *Lemma 2.3.* [1] (The suboptimal overall mixed $\mathrm{H_2/H_\infty}$ static state feedback control problem):

Consider the overall system described by (1). Static state feedback control law (4) satisfies the mixed $\mathbf{H_2/H_\infty}$ control problem if and only if the following LMIs for $\mathbf{Q = Q^T, T, Z}$ and a given positive scalar $\boldsymbol{\gamma > 0}$ are satisfied:

$$\begin{aligned} &Min\ \nu \\ &\text{Subject to: (12) and (14)} \end{aligned} \tag{15}$$

By solving (15), we can find Q, T, Z and ν, with control law (4) computed from (13).

3. GA-based Supervisor

The approach suggested in this paper for solving the mixed H_2/H_∞ control problem of a linear singular perturbation system is different from previous approaches. In this approach, at first an overall linear singular perturbation system is considered and decomposed into a slow and a fast subsystem. Then the mixed H_2/H_∞ control problem is solved for each slow and fast subsystem and K_{fast} and K_{slow} are calculated by solving the corresponding LMIs. It is well known that a fast subsystem can be a good approximation of the transient mode of an overall system response while a slow subsystem can be a good model for approximating the steady state mode of an overall system response. Therefore, the fast subsystem controller K_{fast} can be utilized during the transient time while the slow subsystem controller K_{slow} can be used during the steady state. The control actions of the proposed control scheme are combined by means of a weighting factor with$\alpha \in [0\ 1]$ representing the output of a GA-based supervisor that takes the tracking error e and its time derivatives $\dot{e}, \ddot{e}, \ldots, e^{n-1}$ as inputs. Genetic algorithms have been inspired by Darwin's theory of evolution. The solution to a problem solved by genetic algorithms is provided by an evolutionary process. The algorithm begins with a set of solutions (represented by chromosomes) called population. This is motivated by the hope that the new population will be better than the old one. Solutions are then selected to form new solutions (offspring). This is repeated until some condition (for example number of populations or an improvement in the best solution) is satisfied. The problem solving process can often be expressed as looking for an extreme of a function defined over the search space while in general the GA algorithm tries to find a minimum of the H_2 and H_∞ norms.

The main advantage of GAs is their property of parallelism. GAs are traveling in a search space using more individuals (and with genotype rather than phenotype) so that they are less likely to get stuck in a local extreme if compared to other methods. Moreover, a GA can be implemented easily. Once the basic GA algorithm is formulated, a new chromosome needs to be defined (just one object) to solve another problem. With the same encoding, only a change in the fitness function is needed. However, for some problems, choosing and implementing the encoding and the fitness function can be quite difficult. The disadvantage of GAs is their computational time as GAs may be slower than other methods. The outline of a basic GA is as follows:

1-[Start]: if the length of time interval is $(r+1)T$, each chromosome has r genes. The i-th gene on the chromosome is the value of the weighted coefficient α in the time interval$(i-1)T \le t \le iT$. Encoding heavily depends on the problem's type and complexity. Direct value encoding can be utilized in problems where more complicated values such as real numbers are used. In general, the utilization of binary encoding for this type of problems would be difficult. Here instead, value encoding is adopted and every chromosome is a sequence of real values. Finally, a random population of n chromosomes is generated while some researches show that the best population size (n) depends on the size of the encoded string (chromosomes).

2. **[New population]** Create a new population by repeating the following steps until the new population is complete:

2-1. **[Crossover]** Cross over any two parent chromosomes to form new offspring (children) according to:

$$Child = \alpha \times parent_1 + (1-\alpha) \times parent_2 \tag{16}$$

where α is a random number (the crossover probability is equal to 1).

2-2. **[Mutation]** Mutation generally prevents the GA from falling into local extremes. Mutate new offspring at each gene (positions in chromosomes) according to:

Choose $n \in \{1,2\}$

If $n=0$

$$P_k^{new} = P_k^{old} + \left(upper\ bound - P_k^{old}\right) \times f(t)$$

If $n=1$

$$P_k^{new} = P_k^{old} - \left(Lower\ bound + P_k^{old}\right) \times f(t)$$

where

$$f(t) = \left[rand \times \left(1-\frac{g}{G}\right)\right]^b \tag{17}$$

$b \cong 3$ (the mutation probability is equal to 1).

In the above equation, $P_k$is the k-th gene on the chromosome for which mutation has been utilized. The upper bound and the lower bound are the maximum and minimum values of α (in this paper they are 0 and 1). g is the population number and G is the maximum number of the population in the GA.

3. **[Accepting and Replace]** Add the new offspring to the old population. Sort the new population based on their fitness which is evaluated according to:

If $\left(H_{2_{overal}} < H_{2_{switch}}\right)$ or $\left(H_{\infty_{overal}} < H_{\infty_{switch}}\right)$

$$fitness = \infty$$

otherwise

$$fitness = \left(\frac{H_{2_{switch}}}{H_{2_{overal}}}\right) + \left(\frac{H_{\infty_{switch}}}{H_{\infty_{overal}}}\right)$$

end

Choose n chromosomes with the best fitness as the new population and remove the rest.

5. **[Test]** If the end condition is satisfied, **stop** and return the best solution to the current population.

6. **[Loop]** Go to step **2**

When the norm of the tracking error e and its time derivatives $\dot{e}, \ddot{e}, \dots, e^{n-1}$ are small, the plant is governed by the slow subsystem controller K_{slow} ($\alpha = 1$). Conversely, if the error and its derivatives are large, the plant is governed by the fast subsystem controller K_{fast} ($\alpha = 0$). The control action u is determined by:

$$u = \alpha u_{slow} + (1-\alpha) u_{fast} \tag{18}$$

where:

$$\begin{aligned} u_{slow} &= K_{slow} x_1 \\ u_{fast} &= K_{fast} x_2 \end{aligned} \tag{19}$$

The structure of the proposed control scheme with a GA-based supervisor is depicted in Figure 1.

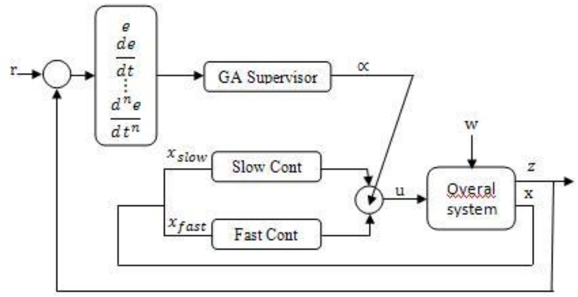

Fig. 1. Structure of the proposed controller.

4. Stability Analysis

The theorem of Essounbouli et al. [7] is utilized to prove the global stability of the system governed by the control law in (18). Similar to [7], this theorem is rewritten as follows:

Theorem 4.1. Consider a combined GA-based logic control system as described in this work. If

1. There is a positive definite, continuously differentiable and radially unbounded scalar function V for each subsystem,

2. Every GA subsystem gives a negative definite $\dot{V}$ in its active region,

3. The weighted sum method is utilized such that for any control input u:

Table 1

Results of example 5.1.

	H_2	H_∞
Overall	11.9754	1.8664
GA-Switching	0.2082	0.6983
Fuzzy Switching[17]	2.9774	1.0274

$$\min(u_{slow}, u_{fast}) \le u \le \max(u_{slow}, u_{fast})$$

, the resulting control u provided by (18) guarantees the global stability of the closed loop system.

Proof: Satisfying the first two conditions guarantees the existence of a Lyapunov function in the active region. It is a sufficient condition for ensuring the asymptotic stability of the system during the transition from the fast subsystem controller to the slow subsystem controller. Consider the Lyapunov function $V_{fast} = \xi^T P_{fast} \xi$ where P_{fast} is a positive definite matrix and the solution of (15) for fast subsystem (11). Consider the Lyapunov function $V_{slow} = \xi^T P_{slow} \xi$ where P_{slow} is a positive definite matrix and the solution of (15) for slow subsystem (9). To satisfy the second condition of the theorem, it suffices to choose P_{fast} and P_{slow} such that:

$$P_{slow} \le P_{fast} \tag{20}$$

This condition guarantees that in the neighborhood of the steady state (the slow subsystem controller), the value of the Lyapunov function V_{fast} is greater than that of V_{slow}. To satisfy the third condition, the balancing term α takes its values in the interval [0 1]. Consequently, the three conditions of the above theorem are satisfied and the global stability of the system is guaranteed. Thus, the problem formulation (the switching H_2/H_∞ control) can be formulated as:

$$\begin{aligned} &\text{Minimize } (\|T(s,K)\|_2)_{slow} \\ &\text{subject to: } (\|T(s,K)\|_\infty)_{slow} < \gamma_{slow} \\ \text{and } &\text{Minimize } (\|T(s,K)\|_2)_{fast} \\ &\text{subject to: } (\|T(s,K)\|_\infty)_{fast} < \gamma_{fast} \end{aligned} \tag{21}$$

while: $P_{slow} \le P_{fast}$

5. Design Procedure

The design procedure can be summarized as follows. Compute the slow and fast subsystems of the overall system in (1) from (9) and (11). Solve the control problem (21) for both the slow and fast subsystems defined in (9) and (11) with given positive scalars γ_{slow} and γ_{fast} to find K_{slow} and K_{fast} from (13). Compute u_{slow} and u_{fast} from (19). Calculate the overall control signal u from $u = \alpha u_{slow} + (1-\alpha) u_{fast}$ where $\alpha \in [0 \quad 1]$ is governed by the GA-based supervisor according to the corresponding error and its derivatives. Apply this control signal to (1) and construct the closed loop system in (5).

Example 5.1. Here, to demonstrate the solubility of various LMIs, the simplicity and the low conservatism of the proposed method, a fourth-order, four-output, two-input example is being considered and a switching static state feedback controller is sought. Consider a singularly perturbated system described by (1) with:

$$A_1 = \begin{bmatrix} -3 & 1 \\ 1 & -1 \end{bmatrix},\ A_2 = \begin{bmatrix} 1.5 & 1 \\ 1 & 1 \end{bmatrix},\ A_3 = \begin{bmatrix} 0.5 & 1 \\ 1 & 2 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} -5 & 1 \\ 3 & -4 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\ B_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B_{w_1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, B_{w_2} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C_{z_1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},\ C_{z_2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},\ D_z = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix},\ C_2 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \varepsilon = 0.1$$

Following the proposed design method in section 5, these results are obtained:

$$K_{slow} = \begin{bmatrix} -48.54 & -109.06 \\ -39.24 & -88.19 \end{bmatrix}$$

$$K_{fast} = 10^{-11} \times \begin{bmatrix} 0.07 & -0.005 \\ -0.33 & -0.21 \end{bmatrix}$$

$$K_{overal} = \begin{bmatrix} -54.72 & -122.82 & -7.63 & -6.18 \\ -44.29 & -99.43 & -6.18 & -5 \end{bmatrix}$$

$$K_{switching} = [\alpha K_{slow} \quad (1-\alpha) K_{fast}]$$

while the responses of the system states are presented in Figure 2 for an arbitrary excitation random signal [17]. We obtain K_{overal} by solving mixed H_2/H_∞ control problem (15) for full order system (1). K_{slow} and K_{fast} are calculated by solving control problem (21) for both slow and fast subsystems (9) and (11).

Based on the simulation results of example 5.1 in Table 1, it is clear that the suggested method gives a better response than the conventional overall design method. Moreover, the results of this method are better than those of the fuzzy supervisor-based switching approach [17]. In our proposed switching method, with a smaller γ for the H_∞constraint, we have a smaller H_2 norm. But both of H_2 and H_∞ norms increase in the conventional overall method. Figure 2 also shows that the state regulation in the proposed controller is better related to the conventional overall controller.

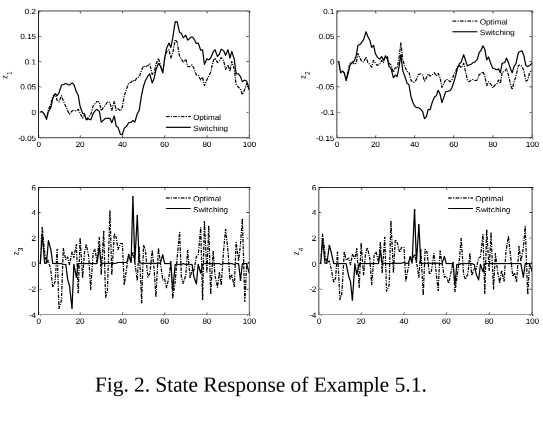

Fig. 2. State Response of Example 5.1.

6. Conclusion

In this paper, a convex optimization method is used to design a logic-based switching H_2/H_∞ controller for a linear singular perturbation system. The proposed controller scheme guarantees the stability of the closed loop system and satisfies the prescribed level of performance indexes for both H_2 and H_∞ norms. Using two reduced-order fast and slow mode controllers instead of one full-order overall controller is the main contribution of this study. The suggested GA-based supervisor scheme is able to manage both fast and slow controller performances efficiently. In reality, the fast mode controller has a good performance in the transient mode (fast dynamic response and low energy impulse response) and the slow mode controller affects the steady state section and is able to attenuate the interaction of low frequency disturbances. The simulation results presented indicate that the proposed novel control scheme results in a considerable improvement in the performance of the closed loop systems.

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