

An Adaptive Approach to Increase Accuracy of Forward Algorithm for Solving Evaluation Problems on Unstable Statistical Data Set

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Abstract

Nowadays, Hidden Markov models are extensively utilized for modeling stochastic processes. These models help researchers establish and implement the desired theoretical foundations using Markov algorithms such as Forward one. However, Using Stability hypothesis and the mean statistic for determining the values of Markov functions on unstable statistical data set has led to a significant reduction in the accuracy of Markov algorithms including Forward algorithm used in solving Evaluation problems.

The model's parameters such as the occurrence probability of observation symbol being produced by state, varies directly among the successive events. Since the probability value of the above-mentioned parameter plays an important role in the accurate Evaluation and assessment of the probability of observations' occurrence in the Evaluation problem by Forward algorithm, the variations between events and observations generated by the States should be automatically extracted. In order to achieve this, the current paper proposes an adaptive parameter for event probability in order to match and adjust the variations in the parameter after each event during the lifetime of Forward algorithm. The results of the experiments on a real set of data indicates the superior performance of the proposed method compared to other conventional methods regarding their accuracy.

Keywords: Hidden Markov model, Evaluation problem, Unstable statistical data set, Forward algorithm.

1. Introduction

Establishing a statistical model for modeling unstable statistical data set, is one of the main objectives of different researchers. Markov modeling is one of the most popular methods for modeling stochastic processes. The Bayesian and flexible network structure of this model is the reason for its widespread use and popularity.

After the introduction of hidden Markov models in the late 1960s by Baum & Argon [1], utilizing

extended Markov models such as hierarchical hidden Markov model and Factorial hidden Markov model [2] gained much acceptance. Extended hidden Markov models, including Factorial and Hierarchical [3] hidden Markov models use further Bayesian dynamic structures to improve the accuracy of statistical data modeling. Increasing the accuracy for modeling statistical data is intended to improve the efficiency of Markov algorithms including Forward algorithm based on extended hidden Markov models.

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Recent extended models, such as hidden Markov model, utilize average statistic and static theories in order to calculate the values of the likelihood functions of the events of observations. The above-mentioned theory is used despite the fact that in an unstable statistical data set, the model's parameters such as the probability of the event of observation generated by the state, change directly from one event to the next. Neglecting these variations in the value configuration of the likelihood function in the hidden Markov model will lead to complications in modeling stochastic processes. Yu et al. in [4] tried to segment statistical training data in order to solve the problem of variation in Markov function's values during the lifetime of unstable statistical data. In this method in order to solve problems such as evaluation in Markov algorithms based on the above-mentioned model, firstly it should be determined the set of observations belong to which segment of the statistical training data. The need for this extra information renders solving Markov problems solely using the usual assumptions impossible.

In order to increase the accuracy of Forward algorithm in solving evaluation problems in unstable environments, the current paper discusses an adaptive algorithm which is statistical but non-parametric. Accordingly, at first, the variations of the likelihood function for the event of observation symbol generated by the state, influenced by the event of observation symbol, should be calculated in the statistical training data. The calculation process is performed considering the dependent structure of the above-mentioned function. In the following, the values obtained based on the adaptive parameter are used to compare and update the values of the likelihood function for the event of observation symbol generated by the state after each event.

Considering the importance of the accuracy of the values for the likelihood function of the event of observations in solving evaluation problems by Forward algorithm, adaptive algorithms provide better

performance compared to Forward algorithm. In fact, the adaptive algorithms has been able to utilize the average storage of variations in unstable environments to increase the productivity of Markov algorithm in those environments. However, the presence of relative order in the variations of the event of observation symbols in the lifetime of training and test data is one of the requirements for the success of this algorithm. In the following, Section Two discusses and introduces hidden Markov models as well as extended hidden Markov models. Section Three explains the proposed method. Section Four is dedicated to the experimental results which include implementation, comparison and analysis of the methods. Finally, section Five concludes the study.

2. Literature Review

This section introduces hidden Markov model as well as extended hidden Markov models. Extended hidden Markov models use further Bayesian dynamic structures for increasing the accuracy of modeling unstable statistical data set.

2.1. Unstable Statistical Data Set

In the unstable statistical data set, the time interval between the repeated events of the observation symbol V_k generated by state i constantly varies. The rate of these variations can be regular or irregular. The variations in the probability of the event of observations in the majority of natural event possesses minimum order. Let $\{X_t\}$ be a stochastic process and the function $F_x(X_{t_1} + \tau, \dots, X_{t_k} + \tau)$ be a cumulative likelihood function for the simultaneous distribution of $\{X_t\}$ in $t_1 + \tau, \dots, t_k + \tau$ times, then, if equation (1) holds [5], the $\{X_t\}$ process will be stable for k and t_1, \dots, t_k .

$$F_x(x_{t_1} + \tau, \dots, x_{t_k} + \tau) = F_x(x_{t_1}, \dots, x_{t_k}) \quad (1)$$

$$I \leq t \leq T_{Data}$$

The T_{Data} parameter is the lifetime of statistical data observations. The cumulative function of likelihood [16] is defined over all the observations present in the training statistical data. In the following, the three dimensional cumulative likelihood function, B' , is defined for determining the probability of all the events of the observation symbol V_k generated by state i during the training data as used in equation (2).

$$B' - \{b_i^r(k)\}, b_i^r(k) = P(O_i = V_k | q_i = S_i) \quad (2)$$

Where $1 \leq t \leq occurrence(i,k,r)$

$$1 \leq r \leq p_k^i, \text{ Where, } 1 \leq i \leq N, 1 \leq k \leq M$$

$b_i(k)[7]$, shows the probability of the event of observation symbol, k , in the state, i . The p_k^i parameter is the number of the events of observation symbol V_k generated by state i during the lifetime of the training data. The $occurrence(i,k,r)$ parameter, assuming $1 \leq r \leq p_k^i$, is the time or the turn of the r^{th} event for the observation symbol V_k generated by state i during the lifetime of the training data. As can be seen, all the observation symbols with identical states contribute to the variations of this parameter between two sequential events. If the event of the observations during the lifetime of sports data occur in regular time intervals or the variations of time intervals between the events of observations are negligible, then this effect will be small and negligible. However, if the time interval of repeated events of the observations decrease or increase, the value of the cumulative likelihood function decreases and increases, too. In Figure (1), the cumulative likelihood function, B' is used in order to assess the values of the likelihood function for the event of observation symbol of performing the dunk technique in basketball, $V_k = "Dunk"$, performed by the Iranian team, $b_i^r = Iran(V_k = "Dunk")$, for seven events of the observation symbol, $V_k = "Dunk"$ and $1 \leq r \leq 27$, during the lifetime of the training data related to the basketball match between Iran and Taiwan during FIBA Asia cup in 2012 [6].

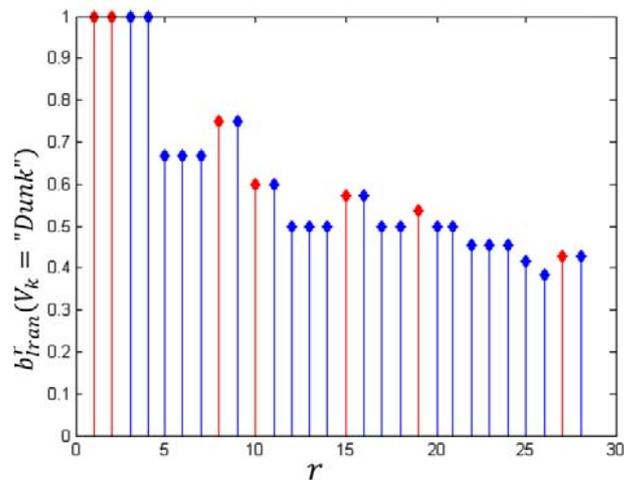


Fig. 1. Cumulative likelihood function, B' , for calculating the probability of the event of observation Symbol $V_k = "Dunk"$ generated by state $i = "Iran"$ during the lifetime of sports data set for the Basketball match between Iran and Taiwan, 2012

By allocating the constant value for the state variable of i , $i = "Iran"$, the three dimensional cumulative likelihood function, B' , will change into a two dimensional function. The parameter r is used to count all the events of the observations symbols in training data. By the event of $V_k = "Dunk"$ during the lifetime of sports data, the value of the parameter $b_i^r = Iran(V_k = "Dunk")$ will be distinguished by the red color. In Figure (1), the reduction of the values of the likelihood function, $b_{Iran}("Dunk")$ with each event of the observation symbol during the lifetime of sports data is visible

2.2. Hidden Markov Model

Hidden Markov model is one of the methods for modeling stochastic processes. This model uses Bayesian dynamic structures for modeling stochastic processes. This characteristic strengthens the mathematical structure in hidden Markov model so that the possibility of implementing a wide variety of theories including speech processing as well as event prediction have been made possible by hidden Markov model. The normal parametric structure of hidden Markov model is defined as $\lambda = (A, B, \pi)$ so that the likelihood function, B , includes $N \times M$ entries

and each entry, $b_i(k)$, shows the probability of the event of observation symbol, K , in the state, j . The likelihood function, A , includes $N \times N$ entries and each entry, a_{ij} , shows the transition probability from state i to state j . Moreover, the likelihood function, π , includes N entries and its entries; namely, $\pi_i, 1 \leq i \leq N$, are the preliminary probabilities of state i . All these parameters are calculated using equations (3), (4), and (5) [7].

$$A = \{a_{ij}\}, \quad a_{ij} = P(q_{t+1} = S_j | q_t = S_i) \quad (3)$$

$$B = \{b_j(k)\}, \quad b_j(k) = P(O_t = V_k | q_t = S_j) \quad (4)$$

$$\pi = \{\pi_i\}, \quad \pi_i = P(q_1 = S_i) \quad (5)$$

$$1 \leq i, j \leq N, \quad 1 \leq K \leq M$$

q_t is the variable of state at time t and O_t is the variable for observation symbol at time t assuming that $1 \leq t \leq T$, the T parameter is the length of the observations of the problem. In order to use the hidden Markov model with the statistical parameters of $\lambda = (A, B, \pi)$, three main issues become apparent. These three issues include evaluation, decoding and learning [7]. According to the structure of each one of these issues, there is an algorithm for solving it. The most important Markov algorithms include Forward algorithm [8] for solving the issue of evaluation, the Viterbi algorithm [9] for solving the decoding problem and the Baum Welch algorithm [10] for solving the learning problem.

2.3. Extended Hidden Markov Model

Extended hidden Markov models are proposed to increase the accuracy of modeling stochastic processes and in turn they increase the accuracy of Markov algorithms such as Forward algorithm in solving problems. Using extended hidden Markov models is carried out based on the requirements and the structure of the statistical data. One of the most important methods which uses Bayesian dynamic structure is Factorial hidden Markov model [11]. In the factorial hidden Markov model, the relation

between states in the lifetime of Markov algorithms is modeled based on the training data. Figure (2) shows a factorial hidden model with the set of states at time t equal to $S_t = \{S_t^1, S_t^2, S_t^3\}$ and the variable of observation at time t equal to Y_t .

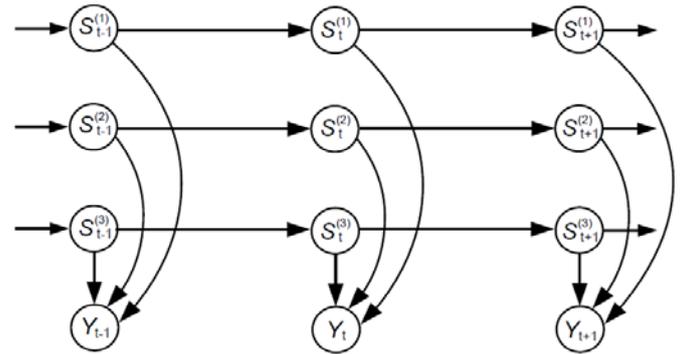


Fig. 2. The structure of Factorial hidden Markov model [11]

As can be seen in Figure (2), the relation between states is modeled based on the previous time interval, $t-1$; current time interval, t and the next time interval, $t+1$.

Using the new Bayesian Probability functions is one of the important ways to develop the Hidden Markov Model. The application of the new probability functions increases the modeling accuracy of statistical data relationships. Evidence Feed-Forward Hidden Markov Model [12], is a newly *non-parametric* model for development and accuracy improvement of Hidden Markov Models by applying the transition probability function between observations symbols in all states of the model. With addition of non-parametric probability of transition between observations symbols, C , the general form of Hidden Markov Model is defined as $\lambda = (A, B, C, \pi)$. The equations (3), (4) and (3) [5] show the calculation methods of probability functions of A , B and π .

3. The Proposed Method

Neglecting the variations of the likelihood function for the event of the observation symbol V_k generated by state i , $b_i(k)$, during the lifetime of training data for unstable s in the process of determining the likelihood function of hidden Markov model leads to a decrease in the accuracy of Forward algorithm in solving evaluation problems. Hence, in the proposed algorithm, using the present order in the variations of this function is considered desirable. Using the adaptive approach in Forward algorithm is considered in order to apply the variations created by the observation event, V_k , onto the likelihood function, $b_i(k)$, using the η_k^i parameter in unstable environments. Before explaining the proposed algorithm, we should first discuss the concept of stableness for the likelihood function, $b_i(k)$, in stochastic processes.

3.1. Calculating the Adaptive Parameter, η_k^i

Considering the importance of the $b_i(k)$ function in solving the evaluation problems in Forward algorithm, the average variations of the likelihood function $b_i(k)$ for each event of the observation symbol V_k during the lifetime of training data should be automatically extracted. The parameter for average sum of variations in the likelihood function of $b_i(k)$ with each event of V_k during the lifetime of statistical data is shown by η_k^i . As can be seen from equation (6), this parameter is calculated using all the events of the observation symbol V_k generated by state i during the lifetime of the training data.

$$\eta_k^i = \text{Mean} \quad \text{Of} \left((b_i^{r+1}(k) - b_i^r(k)) \right) = \frac{1}{P_k^i - 1} \times \sum_{r=1}^{P_k^i - k} (b_i^{r+1}(k) - b_i^r(k)) \quad 1 \leq i \leq N, 1 \leq k \leq M \quad (6)$$

The M parameter is the number of observation symbols, and N is the number of states. P_k^i is equal to the number of events for the observation symbol V_k generated by state i in the training data.

3.2. Using the Adaptive Approach in Forward Algorithm

The adaptive approach is used to apply the effects arising from the event of observation symbol, V_k , on the variations of the likelihood function, $b_i(k)$. This likelihood function is updated with each event of V_k during the lifetime of Forward algorithm using the η_k^i parameter. The process of updating the Forward algorithm is carried out based on the order of retrieving observations by the auxiliary variables in the Forward algorithm as well as the requirements

3.2.1. Adaptive Forward Algorithm

adaptive Forward algorithm is developed in order to update the values of the function $b_i(k)$ by each retrieval of the observation symbol, V_k , by the auxiliary variable of a at time t , a_t . The auxiliary variable of a is a recursive Forward variable. Before using the updating function, the retrieval order in the set of variables in the Forward algorithm should first be considered using the auxiliary variable a . The retrieval order in the set of observations based on the auxiliary variable of a at time t is shown in Table 1.

Table 1

The retrieval order of observations in the Evaluation problem by the auxiliary variable a in the Forward algorithm

Current Time	Called Auxiliary Variables	Called Observation
t=0	$a_1(j) = 0, 1 \leq j \leq N$	-
t=1	$a_1(j) = \pi_j b_j(O_1)$	O_1
t=2	$a_2(j) = b_j(O_2) \sum_{i=1}^N a_1^{(i)} a_{ij}$	O_2
.....
t=T	$a_T(j) = b_j(O_T) \sum_{i=1}^N a_{T-1}^{(i)} a_{ij}$	O_T

The adaptive Forward algorithm is obtained by combining Forward algorithm based on hidden Markov model and the updating function for the function $b_i(k)$. In the following, Forward algorithm

based on the hidden Markov model is shown as algorithm (1).

Algorithm 1. Adaptive Forward algorithm based on hidden Markov model

1: Initialize:
 2: For ($i=1:i \leq N; i++$)
 3: $a_{1(i)} = \pi_{j b_j}(o_1), \Psi_1(j) = 0$
 4: End
 5: Update(1, O_1), $1 \leq j \leq N$
 6: Induction
 7: For($t=2; t \leq T; t=t+1$)
 8: For ($i=1: i \leq N; i++$)
 9: $a_t(i) = b_i(O_t) \sum_{j=1}^N a_{t-1}(j) a_{ij}$
 10: End;
 11: Update(t, O_t)
 12: End
 13: Termination:
 14: For ($i=1: i \leq N; i++$)
 15: $P(O|\lambda) = \sum_{i=1}^N a_T(i)$
 16: End

With each retrieval of observation O_t by the auxiliary variable, a_t , $1 \leq t \leq T$, the function $Update(t, O_t)$ updates the value of the likelihood function, $b_i(k)$. The updating function, $Update(t, O_t)$ is presented in algorithm (2) below.

Algorithm 2. Update algorithm

1: Function Update (O_t)
 2: $O_t = k'$
 3: For($i=1; i \leq N; i=i+1$)
 4: For ($k=1; k \leq M; k=k+1$)
 5: if ($k=k'$) $b_i^{Adjusted}(k') = b_i(k) + \eta_k^i$;
 6: Else $b_i^{Adjusted}(k) = b_i(k)$; End;
 7: $Sum(i, k) = b_i^{Adjusted}(k) + Sum(i, k)$;
 8: End;
 9: End
 10: For($i=1; i \leq N; i=i+1$)
 11: For ($k=1; k \leq M; k=k+1$)
 12: if ($Sum(i, k) \leq 1$) $b_i^{Adjusted}(k) =$

$$\frac{1}{Sum(i, k)} \times b_i^{Adjusted}(k);$$

 13: Else $b_i^{Adjusted}(k) =$

$$\frac{Sum(i)}{1} \times b_i^{Adjusted}(k); End ;$$

 14: End;
 15: End;
 16: End;
 17: End;

The parameter $Sum(i, k)$ is used to count the sum of values of likelihood function, $b_i(k)$ assuming that $1 \leq k \leq M$. The parameter M indicates the number of observation symbols. At the end of algorithm 2, it is necessary to evaluate the necessary requirements in equations (7) and (8) for the adapted values of the function $b_i(k)$; namely, $b_i^{Adjusted}(k)$.

$$b_i^{Adjusted}(k) \geq 0 \quad (7)$$

$$\sum_{k=1}^m b_i^{Adjusted}(k) = 1 \quad (8)$$

$$1 \leq k \leq M, 1 \leq i \leq N$$

At the end of algorithm 1, lines 10 to 16, applying the normalization process will lead to the establishment of the requirements of equations (7) and (8) for the updated values of the likelihood function, $b_i(k)$; namely, $b_i^{Adjusted}(k)$.

4. Results of the Experiments

In the section for the results of the experiments, the accuracy of adaptive Forward algorithm in solving the evaluation problem is compared to the Forward algorithm based on hidden Markov model as well as extended hidden Markov model under unstable statistical data set.

4.1. The Set of Test Data in Experiments

This study tries to use popular stochastic processes such as sport processes. The selected sports data involve the statistical data for basketball matches. Desired training and test set, consists of two professional basketball match. First training and test set used in the experiments have been chosen From the Eighth-week games of professional basketball league of America, NBA, in 2015-16(February) [13]. Considering the teams present in the Evaluation problem and by prioritizing in alphabetical order, the selected states are as follows:

- 1- Denver Nuggets
- 2- Utah Jazz

In the first series of tests, basketball shots techniques are evaluated. Hence, observation symbols include 2-point and 3-point shots. Categorization of observation symbols is carried out based on basketball rules [14]. The list of all the defined observation symbols is as follows:

- 1- Successful 3-point shots (Success 3P)
- 2- Failed 3-point shots (Failed 3P)
- 3- Successful 2-point shots (Success 2P)
- 4- Failed 2-point shots (Failed 2P)

The training statistical data include the ordered pairs of observation symbol and the state generating the observation symbols for the basketball match between Denver and Utah in the second week of 2015-16 league.

The second sport processes used in the experiments have been chosen From the Fifteenth-week games of NBA in 2015-16(March) [13]. By considering the teams present in the problem environment and by prioritizing Based on alphabetical order, the selected states are as follows:

- 1- Denver Nuggets
- 2- Memphis Grizzlies

Observation symbols of both statistical data set are same.

4.2. Experiments' Parameters

Establishing variety in the values of problem parameters including the length of the Evaluation problem and changing the conditions of the test data such as erratic variations in the likelihood function, $b_i(k)$, during the lifetime of the stochastic processes, aim to learn more about the performance of the proposed algorithms in unstable statistical data set.

4.2.1. Percentage of Erratic Changes in the Likelihood Function, $b_i(k)$.

This study utilizes the percentage of erratic changes in the values of likelihood function, $b_i(k)$, with each event of the observation, V_k , during the lifetime of statistical data, SVD_k^i [15]. In function SVD_k^i , the extent of stochastic changes during the lifetime of statistical data, is allocated a value between 0 and 1. When there is Stability in the ratio of generating observation symbol, V_k generated by the state i during the lifetime of statistical data, the value of SVD_k^i will be equal to 0. If the increasing and decreasing steps of the function $b_i(k)$ with each event of V_k are equal, the function SVD_k^i will be equal to 1.

Table 2

The value of percentage of erratic Changes in the Likelihood Function, $b_i(k)$ with each event of observation, V_k , during the Lifetime of Test data, SVD_k^i .

index	Match Name	SVD_k^i
1	Denver Vs Utah Jazz	0.141
2	Denver Vs Memphis	0.337

As can be seen in Table 2, of the Sport data set (1), involve much more erratic changes in the values of the likelihood function $b_i(k)$ at the event of observation symbol, V_k , during the life of the test data. The results presented in Table 2 can be useful for comparing the performances of the proposed algorithm and Forward algorithm under unstable statistical data set.

4.2.2. The Length of the Evaluation Problem

According to equation (1), for $L = 1$ and $L=2$ of the observation sequence, $o = \{o_1, \dots, o_T\}$, the impact of initial state probabilities in determining the value of observation symbol probability k produced by state i , $b_i(k)$, is respectively 100% and 50%. So, it is assumed that the minimum length of decoding equals to $L=3$. To reduce the role of probability function π and therefore correctly evaluate of Hidden Markov algorithms performance, the Evaluation length should be properly selected in order to accurately estimate of state probability of evaluation problems. On the other hand, increasing the length of evaluation problems leads to increase the number solution paths exponentially. Hence, the evaluation problem choices with a high length (above 6) are avoided. Hereupon, the length of the Evaluation problems selected 3, 4, 5, and 6, in each of the four experiments respectively.

4.3. Accuracy Percentage of Adaptive Forward Algorithm in Experiments

The accuracy percentage of Forward algorithm is determined based on the average accuracy of

calculating the probability of sequential event of observations, $o = \{o_1, \dots, o_T\}$, for the given model of $\lambda, P(O|\lambda)$, by the Forward algorithm. In the following, the obtained results will be analyzed considering the extent of erratic variations in the likelihood function, $b_i(k)$, during the lifetime of test data and the length of the Evaluation problem, T .

In the following, accuracy percentage of the adaptive Forward algorithm based on the hidden Markov model, *AF HMM*, will be compared to the average accuracy of the Forward algorithm based on the hidden Markov model [7], *F HMM*, the factorial hidden Markov model [11], *F FHMM*, and Evidence Feed Forward hidden Markov model [12], *F EFFHMM*. In order to evaluate the performance of the proposed algorithm in different data set, the erratic processes have been chosen from professional sport statistical data set.

A. Accuracy Percentage of the Adaptive Forward Algorithm for Sport Data Set (1)

The Relative high value of the variations in the likelihood function, $b_i(k)$, at the event of observation V_k or SVD_k^i is a characteristic of the Sport data set (1). In Figure (3), the average accuracy of the adaptive Forward algorithm based on hidden Markov model, the Forward algorithm based on hidden Markov model, the Evidence Feed Forward hidden Markov model and the factorial hidden Markov model for the accurate calculation of the probability of event, $P(O|\lambda)$, over of the Sport data set (1) test data are presented.

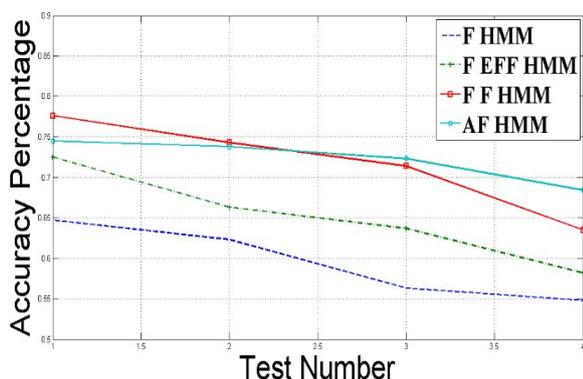


Fig. 3. The Accuracy of the adaptive Forward algorithm and the Forward algorithm based on the extended hidden Markov models in solving Evaluation problems for sport data set (1)

Increasing the length of the Evaluation problem and the numerous observation events during the lifetime of the Forward algorithm will lead to a decrease in the average accuracy of the Forward algorithm in solving the Evaluation problem. However, the adaptive Forward algorithm significantly prevents the reduction of the accuracy of the algorithm by adjusting the probability value of $b_i(k)$ and applying the variations rising from observation events in the Evaluation problem on the values of this function.

B. Accuracy percentage of the Adaptive Forward Algorithm for Sport Data Set (2)

The average accuracy of the adaptive Forward algorithm, Forward algorithm based on hidden Markov model, Forward algorithm based on factorial hidden Markov model, and Forward algorithm based on Evidence Feed Forward hidden Markov model for the selected sports processes are presented in Figure (4).

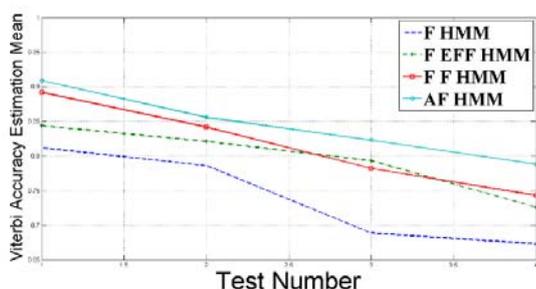


Fig. 4. The accuracy of the adaptive Forward algorithm and the Forward algorithm based on the extended hidden Markov models in solving Evaluation problems for sport data set (2)

Decreased erratic variations of the likelihood function, $b_i(k)$, at the event of the observation symbol, V_k , during the lifetime of the selected statistical sports data will lead to an increase in the accuracy of the adaptive Forward algorithm in solving Evaluation problems compared to the Forward algorithm based on the factorial and Evidence Feed Forward hidden Markov models. In the following, the Wilcoxon signed-rank test is used for analyzing the hypothesis positing the presence of a significant difference between the accuracy values of adaptive and non-adaptive Forward algorithm for the experiments in Sections 4.3 and 4.4 with the confidence level of 95% (alpha coefficient= 0.05). The result of the Wilcoxon signed-rank test for assessing the above-mentioned hypothesis with 95% confidence level is presented in Table 4.

Table 3

Wilcoxon signed-rank test over the accuracy values of adaptive Forward algorithm and Forward algorithm based on extended hidden Markov models

Algorithm	F HMM	F EFFHMM	F FHMM	A FHMM
F HMM	-	-	-	-
F EFF HMM	0.043	-	-	-
F FHMM	0.041	0.047	-	-
AF HMM	0.041	0.047	0.143	-

The values lower than the alpha level, of $\alpha=0.05$, for the values of the Wilcoxon signed-rank test in Table 3 indicate the significant difference for the Evaluation accuracy of compared algorithms. Proving the presence of a significant difference between the accuracy of the adaptive Forward algorithm and the Forward algorithm based on hidden Markov model and extended hidden Markov models is one of the important results of this test.

4.4. Comparing the Order of Complexity for the Calculations

Selecting the appropriate algorithm for the implementation of the theoretical foundations should be done based on the problem's conditions, the accuracy required, and the resource limitations such as time and the extent of calculations. The limited number of observation symbols and short-length

Evaluation problems diminish the influence of the resources. However, by increasing the length of the problem and the number of the observation symbols, the decision making conditions will definitely change. In the adaptive Forward algorithm, an extra nested ring is used for adjusting the values of the likelihood function for the event of the observations generated by the states. Hence, the complexity of the calculations for the adaptive Forward algorithm based on the hidden Markov model is as follows.

$$o_{AF\ HMM} = o((N^2 \times N)T) = o(N^3T) \quad (9)$$

In the following, the order of complexity for the adaptive Forward algorithm based on hidden Markov model, $AF\ HMM$, the Forward algorithm based on factorial hidden Markov model, $F\ FHMM$, the Forward algorithm based on Evidence Feed Forward hidden Markov model, $F\ EFFHMM$, and the Forward algorithm based on hidden Markov model, $F\ HMM$, are presented in Table 4.

Table 4

Order of complexity of calculations for the adaptive Forward algorithm and Forward algorithm based on extended hidden Markov models [16]

Algorithm	Complexity Order
$F\ HMM$	$O(N^2T)$
$F\ FHMM$	$O(N^3T)$
$F\ EFFHMM$	$O(N^2T)$
$AF\ HMM$	$O(N^3T)$

Table 4, indicates the higher order of complexity of calculations for the adaptive Forward algorithm, and the Forward algorithm based on factorial hidden Markov model. By increasing the length of the Evaluation problem or the number of observation symbols, the significance of the calculation complexity in selecting the proposed algorithm also increases.

5. Conclusions

In this study, an adaptive approach for the likelihood function, $b_i(k)$, at the event of the

observations for the Evaluation problem during the lifetime of the Forward algorithm under unstable statistical data set proposed. Updating the $b_i(k)$ function using the parameter for average variations for the values of the $b_i(k)$ function, at the event of V_k will lead to an increase in the accuracy of Forward algorithm under unstable statistical data set. The capability of the proposed algorithms is considerably related to the percentage of changes in the values of the likelihood function $b_i(k)$ at the event of V_k during the lifetime of the test data, \overline{SVD}_k^j . Accordingly, reducing the value of the above-mentioned parameter will lead to an increase in the accuracy of the algorithm in solving Evaluation problems. Moreover, by increasing the value of this parameter and decreasing the present order in the variations of the $b_i(k)$ function at the event of V_k , in the context of the test data, the accuracy of the adaptive Forward algorithm in solving the Evaluation problems will decrease. Considering the relation between the order of variations in the likelihood function of the event of observations and the value of the adaptive parameters, the proposed algorithm provides an desirable performance under all the test set. The higher order of complexity of calculations in adaptive Forward algorithm compared to the Forward algorithm based on the hidden Markov model is one of the disadvantages of the proposed algorithm.

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