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Uncertain Fuzzy Time Series: Technical and Mathematical Review

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Abstract

Time series consists of a sequence of observations, measured at moments in time, sorted chronologically, and evenly spaced from each other, so the data are usually dependent on each other. Uncertainty is the consequence of imperfection of knowledge about a state or a process. The time series is an important class of time-based data objects and it can be easily obtained from scientific and financial applications. Main carrier of time series forecasting is which constitutes the level of uncertainty human knowledge, with its intrinsic ambiguity and vagueness in complex and non-stationary criteria. In this study, a comprehensive revision on the existing time series pattern analysis research is given. They are generally categorized into representation and indexing, similarity measure, uncertainty modeling, visualization and mining. Various Fuzzy Time Series (FTS) models have been proposed in scientific literature during the past decades or so. Among the most accurate FTS models found in literature are the high order models. However, three fundamental issues need to be resolved with regards to the high order models. The primary objective of this paper is to serve as a glossary for interested researchers to have an overall depiction on the current time series prediction and fuzzy time-series models development.

Keywords: Fuzzy Logic, Time-Series, Uncertainty Modeling

1. Introduction

Real world problems require exploitation of frameworks that enable handling different types and levels of uncertainty. Fuzzy sets enable handling intra- and inter-uncertainties such as uncertainty of a subject and uncertainties among different subjects. Also, a time series is a collection of observations made chronologically. The characteristics of a nonlinear dynamical system within chaotic system is more intensely studied recently, due to many real-world applications of the nonlinear chaotic system are increasing. For characterizing the ordinary system, usually the relationship between the linearity and the nonlinearity of parameters in the system is needed to be firstly derived, however, creating the mathematical model of the real chaotic system is still a problematic since insufficient basic physical phenomena should be analyzed. However, according to the unique behavior of the time series data, existing research is still inadequate There is still room for us to further investigate and develop. For example, while most of the research communities have concentrated on the mining tasks, the fundamental problem on how to represent a time series has not yet been fully addressed so far. To represent a time series is essential, because time series data is hard to manipulate in its original structure. The high dimensionality of time series data creates difficulties in applying existing data analysis techniques to it. Therefore, defining a more effective and efficient time series representation scheme is of

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fundamental importance. The artificial neural networks approach that performed based on nonlinear mathematical model is quite adequate to be used to analyze the chaotic phenomena within the system. Solving the multi-step ahead prediction problem of time series chaotic system is one of the top challenging issues, especially on how to obtain a higher prediction rate. The fuzzy time series prediction area is the motivation and study case for this paper, and particularly several researches have been reviewed, respectfully. The prediction of times series (TS) has played an important role in many science fields of practical application as engineering, biology, physics, meteorology, etc. In particular, and due to their dynamical properties, the analysis and prediction of chaotic time series have been of interest for the science community. In the literature, we found many methods focused on the prediction of chaotic time series, for example, those based on artificial neural network (ANN) models as the back-propagation algorithm. Additionally, time series data, which is characterized by its numerical and continuous nature, is always considered as a whole instead of individual numerical field. The increasing use of time series data has initiated a great deal of research and development attempts in the field of data mining. The abundant research on time series data mining in the last decade could hamper the entry of interested researchers, due to its complexity. There are various kinds of uncertain time series related research, for example, finding similar time series, subsequence searching in time series, dimensionality reduction and handling uncertainty. Those researches have been studied in sizeable detail by both database and pattern recognition societies for different domains of uncertain time series which have been reviewed in next section.

2. Research background

This section presents a brief overview of the Type-1 fuzzy systems. Then it follows by a review of type-2 fuzzy systems (T2FS) concepts and its mathematic definitions.

2.1 Type-1 Fuzzy System

Lotfi Zadeh, as the founder of fuzzy logic, has long been active in system science. He appreciates adequately the contradiction between and unity of accuracy and fuzziness. He realized that a complex system was difficult to be dealt with in the framework of precise mathematics. He analyzed fuzziness, approximation, randomness, and ambiguity. He maintained that fuzziness should become a basis research object, put forward the basic concepts of degree of membership and membership function, and produced the fuzzy set, thus being considered as the creator of fuzzy theory. His paper titled "Fuzzy Sets" was regarded as one of the first works in fuzzy theory. Fuzziness is a characteristic feature of modern science to describe quantitative relationships and space formation by using precise definitions and rigidly proven theorems, and to explore the laws of the objective world by using precisely controlled experimental methods, accurate measurements, and calculation so as to establish a rigorous theoretic system. It was believed that everything should and could be more precise and there was nothing that need not and could not be more precise. If things should go to the contrary, it was because people's understanding of the problems had not reached such a depth.

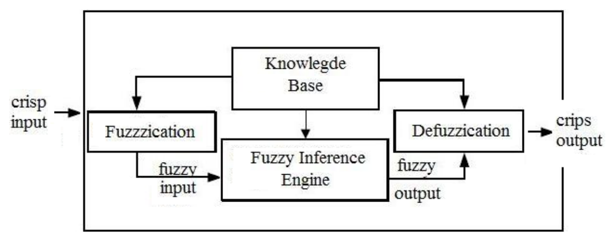

Fig.2. A type-1 Fuzzy System Structure [7]

2.2 Type-2 Fuzzy System

Consider the transition from ordinary sets to fuzzy sets. When the membership of an element in a set cannot be determined as either 0 or 1, type-1 fuzzy sets are used. Similarly, when the circumstances are so uncertain that it is difficult to determine the membership grade even as a crisp number in [0, 1] then fuzzy sets of type 2 can be used, a concept that was first introduced in Zadeh. When something is uncertain (e.g., a measurement), its exact value cannot be determined, so using type-1 sets makes more sense than using crisp sets.

A type-2 fuzzy set, denoted $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}(x,u)}$ where $x \in X$ and $u \in J_x \subseteq [0,1]$, i.e.,

$$\tilde{A} = \left\{ \left((x,u), \mu_{\tilde{A}(x,u)} \right) | \forall x \in X, \forall \in J_x \subseteq [0,1] \right\} \quad (1)$$

In which $0 \leq \mu_{\tilde{A}(x,u)} \leq 1$, X is the domain of fuzzy set and J_x is the domain of the secondary membership function at x. $\tilde{A}$ can be expressed as:

$$\tilde{A} = \frac{\int_{x\in X} \int_{u\in J_x} \mu_{\tilde{A}}(x,u)}{x, uJ_x} \subseteq [0,1] \quad (2)$$

where $\iint$ denotes union over all admissible x and u.

An Interval Type-2 fuzzy system, $\tilde{A}$, is to-date the most widely used kind of T2 FS, and is the only kind of T2 FS that is considered in this paper. It is described as:

$$\tilde{A} = \frac{\int_{x\in X} \int_{u\in J_x} 1}{x,u} = \frac{\int_{x\in X} \left[\frac{\int_{u\in J_x} 1}{u} \right]}{x} \quad (3)$$

where x is the primary variable, J_x , an interval in [0,1], is the primary membership of x, u is the secondary variable, and $\int_{u\in J_x}$ is the secondary membership function (MF) at *x*. Uncertainty about $\tilde{A}$ is conveyed by the union of all of the primary memberships, called the footprint of uncertainty of $\tilde{A}$, $[FOU(\widetilde{A})]$, i.e,

$$FOU(\widetilde{A}) = \cup_{x\in X} J_x \quad (4)$$

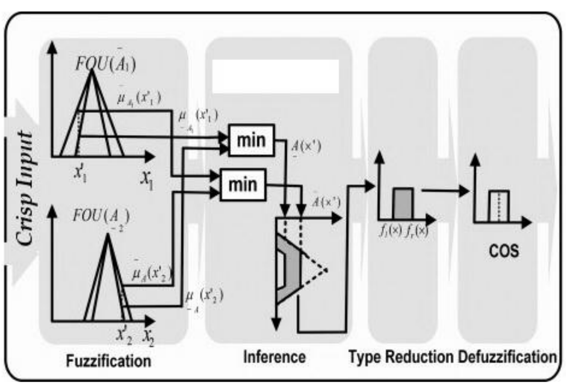

Fig.2 Interval Type-2 Fuzzy System Structure [13]

membership function (UMF) which is called footprint of uncertainty, can handle the uncertainty associated with the system. Let us assume that $J_x \subset [0, 1]$ means the primary membership of element x. FOU of a type-2 fuzzy set A ⊆ X will be a constrained region containing of all the points of primary membership of elements x.

$$\mathrm{FOU(A)} = \bigcup_{x\epsilon X}^{i} \mathrm{Jx} \quad (5)$$

Consider the MFs of the type-2 fuzzy set was described using the Gaussian function with the assumption that a standard deviation σ changes in the interval [σ1, σ2] for μA(X)is:

$$\mathrm{N(m.\sigma; x)} \ \exp\left[-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2\right].\sigma\epsilon[\sigma 1.\sigma 2] \quad (6)$$

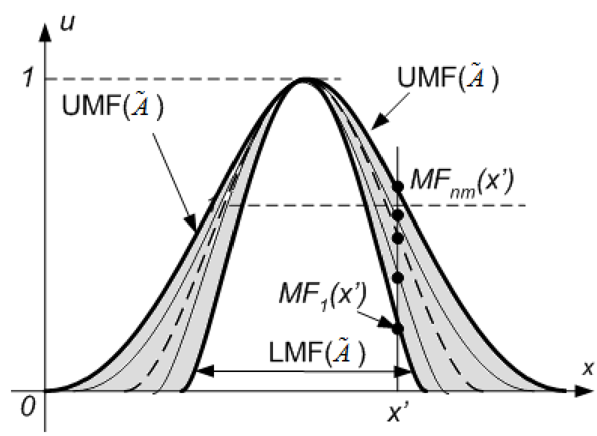

Fig.3 FOU for Gaussian primary membership function

The secondary MF is defined for each point x and the corresponding interval Jx. Figure 3 shows the footprint of uncertainty of the discussed type-2 fuzzy set. The thick solid curve in this figure denotes the Upper Membership Function (UMF), and the thick dashed curve denotes the Lower Membership Function (LMF) where $\{\overline{\sigma}, \underline{\sigma}\}$ and *m* are the uncertain standard deviation set and the mean of Type-2 fuzzy Gaussian membership function, respectively. The value of membership degree can be written as follows:

$$\bar{\mu}_A(x) = \exp\left(-\frac{1}{2}\frac{(x-m)^2}{\bar{\sigma}^2}\right) \quad (7)$$

$$\underline{\mu}_A(x) = \exp\left(-\frac{1}{2}\frac{(x-m)^2}{\underline{\sigma}^2}\right) \quad (8)$$

3. Literature Review

When historical data are linguistic values, the conventional time series is difficult to deal with the forecasting problems. Hence, time-variant and time-invariant fuzzy time series ([1-3]) are first introduced to solve this forecasting problem. Later a relatively simple method by using simple arithmetic operations to modify the max-min composition operations is proposed in [4]. During the past few decades, fuzzy time series forecasting has become a hot topic which gains much more attentions by scholars. In the research of fuzzy time series, how to select the proper intervals and their lengths is an important question. In [5], it is shown that different lengths of intervals may lead to various forecasting results. In order to improve the prediction accuracy, in [6] and [7] particle swarm optimization algorithm is used to adjust the lengths of intervals. In [8] genetic algorithm (GA) is applied to determine the partition of the universe of discourse. Besides, in [9-11] some scholars use the concept of interval information granules to optimal interval length. Another important question is about the variables used in fuzzy time series forecasting. Many traditional fuzzy time series forecasting method only use one variable in the process of modeling, namely Type 1 fuzzy time series forecasting model. The variable used in Type 1 model is also called Type 1 observation. However, for some complex model, the change of a variable value is not only related to its own laws but also affected by other factors. In order to take advantage of the relevant factors, a Type 2 fuzzy time series forecasting model is introduced in [12]. Up to now, several short-term prediction methods have proven to be efficient in the forecast of chaotic time series and therefore in the characterization of dynamical system (for example, see [5, 6 11, 12, 13] and references therein). In contrast, the long-term prediction still requires methods to improve the forecast effectiveness. Furthermore, in the literature the long-term forecast has not been widely studied.

In the context of time series forecasting, the fundamental problem is how to represent the time series and handling the uncertainty of associated with them. Forecasting is a powerful way to reveal and visualize structure of uncertain data. When dealing with time series, selecting a suitable measure to evaluate the similarities/dissimilarities within the data becomes necessary and subsequently it exhibits a significant impact on the results of clustering. This selection should be based upon the nature of time series and the application itself. In this part of paper various approaches have been reviewed, respectively.

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Integration by average: the author ANFIS integration method is the simplest and most straightforward, consists the sum of the results generated by each ANFIS is divided by the sum of number of ANFIS, and the disadvantage is that there are cases in which the prognosis is not good. Integration of weighted average: proposed method is an extension of the integration by average, with the main difference that the weighted average assigns importance weights to each of the ANFIS. These weights are assigned to a particular ANFIS based on several factors; the most important is the knowledge product experience. Their integration method belongs to the well-known aggregation operators.

Soto and Melin in [19] describe the Mackey-Glass time series prediction using genetic optimization of type-1 and interval type-2 fuzzy integrators in Ensembles of adaptive neuro-fuzzy inferences systems (ANFIS) models, with emphasis on its application to the prediction of chaotic time series. The considered chaotic problem is the Mackey-Glass time series that is generated from the differential equations, so their benchmark time series is used to the test of performance of the proposed Ensemble architecture. They used the interval type-2 and type-1 fuzzy systems to integrate the outputs (forecasts) of each of the ANFIS models in the Ensemble. Genetic algorithms (GAs) were used for the optimization of memberships function (with linguistic labels "Small, Middle, and Large") parameters of the fuzzy integrators. In the experiments, the GAs optimized the Gaussians, generalized bell and triangular membership functions for each of the fuzzy integrators, thereby increasing the complexity of the training. Simulation results show the effectiveness of the proposed approach. In [20] Castillo and Soto describe the construction of intelligent hybrid architectures and the optimization of the fuzzy integrators for time series prediction; interval type-2 fuzzy neural networks (IT2FNN). IT2FNN used hybrid learning algorithm techniques (gradient descent back-propagation and gradient descent with adaptive learning rate back-propagation). The IT2FNN is represented by Takagi–Sugeno–Kang reasoning. Therefore, The TSK IT2FNN is represented as an adaptive neural network with hybrid learning in order to automatically generate an interval type-2 fuzzy logic system (TSK IT2FLS). They use interval type-2 and type-1 fuzzy systems to integrate the output (forecast) of each Ensemble of ANFIS models. Particle Swarm Optimization (PSO) was used for the optimization of membership functions (MFs) parameters of the fuzzy integrators. The Mackey-Glass time series is used to test of performance of the proposed architecture. Simulation results show the effectiveness of the proposed approach.

3.1 Fuzzifying Historical Data

The fuzzification algorithm (FA) proposed here generates a series of trapezoidal fuzzy sets from a given dataset and establishes associations between the values in the dataset and the fuzzy sets generated. It is inspired by the trapezoid fuzzification approach proposed by Cheng et al in [14]. They introduced an approach where the crisp intervals, generally defined by the user at the initial step of FTS, are replaced with trapezoidal fuzzy sets with overlapping boundaries. This overlap implies that a value may belong to more than one set. If a value belongs to more than one set, it is associated to the set where its degree of membership is highest. The FA introduced here follows the same principles but differs from the approach described by Cheng et al [14] by performing automatically the calculation of the fuzzy intervals/sets. The fuzzification approach published in [14], requires the user to specify the number of sets. This is an undesirable requirement in situations where multiple forecasting problems need to be solved. For example, a grocery store may need forecast information related to thousands of products. The proposed algorithm aims to solve this problem by determining the number of sets on basis of the variations in data. Another aspect this algorithm attempts to capture, is the notion of a non-static universe set. Whenever values are encountered which fall outside the boundaries of the current universe set, the universe set has to augment accordingly. This aspect, in particular, has not received much attention in current publications. The most likely reason for this is that current modalities rely on the assumption of predetermined outcomes, and therefore, no revisions of the universe set are required. In real life situations though, future outcomes are rarely known. The basic idea of the algorithm described in the following paragraphs, is to repeat the fuzzification procedure when the dataset is updated. The proposed procedure can be described as a six-step process:

Step 1: Sort the values in the current dataset in ascending order.
Step 2: Compute the average distance between any two consecutive values in the sorted dataset and the corresponding standard deviation.
Step 3: Eliminate outliers from the sorted dataset.
Step 4: Compute the revised average distance between any two remaining consecutive values in the sorted dataset.
Step 5: Define the universe of discourse.
Step 6: Fuzzify the dataset using the trapezoid fuzzification approach

First the values in the historical dataset are sorted in ascending order. Then the average distance between any two consecutive values in the sorted dataset is computed and the corresponding standard deviation. Both the average distance and standard deviation are used in step 3 to define outliers in the sorted dataset.
Outliers are values which are either abnormally high or abnormally low. These are eliminated from the sorted dataset, because the intention here is to obtain an average distance value free of distortions. An outlier, in this context, is defined as a value less than or larger than one standard deviation from average. After the elimination process is completed, a revised average distance value is computed for the remaining values in the sorted dataset, as in step 2. The revised average distance, obtained in step 4, is used in step 5 and 6 to partition the universe of discourse into a series of trapezoidal fuzzy sets. Basically, the intention is to create a series of trapezoidal approximations which capture the generic nature of data as closely as possible, in the sense that we neither want the spread of individual functions to be to narrow or to wide. In step 5, the universe of discourse is determined. Its lower and upper bound is determined by locating the largest and lowest values in the dataset and augment these by:

(1) subtracting the revised average distance from the lowest value and
(2) adding the revised average distance to the highest value.

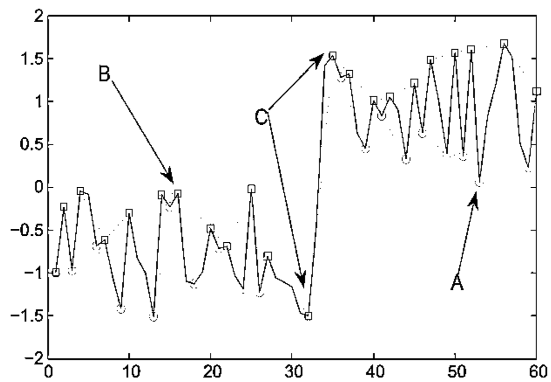

Fig.4 Fuzzy Time-Series: The upper bound and lower bound of time series. The points marked 'A' represent the lower bound points, the points marked 'B' mean the upper bound points and the 'C' denote upper bound point and lower bound point together [28]

In [29] they proposed a new forecasting approach based on fuzzy time series (FTS) that takes advantage of fuzzy and stochastic patterns on data and is capable to deal with point, interval, and distribution forecasts. The method proposed was empirically tested with typical financial time series, and the results were compared with other standard FTS and statistical methods. The results show that the proposed method obtained accurate results and outperformed standard FTS methods. The proposed method also combines versatility, scalability, and low computational cost, making it useful on a wide range of application scenarios.

3.2 Evolutionary Fuzzy-Time Series

PSO is an optimization technique applicable to continuous non-linear functions. It was first introduced in [19]. The algorithm simulates the social behaviors shown by various kinds of organisms such as bird flocking or fish schooling. Imagine a group of birds randomly foraging in an area. The group shares the common goal of locating a single piece food. While foraging, individual birds may learn from the discoveries and past experiences of other birds through social interaction. Each bird synchronizes its movements with group while simultaneously avoiding collisions with other birds. As the search continues, the birds move closer toward the place where the food is by following the bird which is closest to the food. In PSO, bird flocks are represented as particle swarms searching for the best solution in a virtual search space. A fitness value is associated to each particle which is evaluated against a fitness function to be optimized, and the movement of each particle is directed by a velocity parameter. During each iteration, particles move about randomly within a limited area, but individual particle movement is directed toward the particle which is closest to the optimal solution. Each particle remembers its personal best position (the best position found by the particle itself) as well as the global best position (the best solution found by any particle in the group). The parameters are updated each time another best position is found. This way, the solution evolves as each particle moves about. Compared to other related approaches such as genetic algorithms and neural networks, PSO it is quite simple and easy to implement. It is initialized with a set of randomly generated particles which in fact are candidate solutions. An iterative search process is then set in motion to improve the set of current solutions. During each iteration, new solutions are proposed by each particle which are individually evaluated against: (1) the particles own personal best solution found in any proceeding iteration and (2) the global best solution currently found by any particle in the swarm.

We refer to each candidate solution as a position. If a particle finds a position better than its current personal best position, its personal best position is updated. Moreover, if the new personal best position is better than the current global best position, the global best position is updated.

3.3 Neuro-Fuzzy Time Series

There exists a diversity of methods of integration or aggregation of information, and we mention some of these methods: *Integration by average*: this method is used in the Ensembles of ANFIS. This integration method is the simplest and most straightforward, consists the sum of the results generated by each ANFIS is divided by the sum of number of ANFIS, and the disadvantage is that there are cases in which the prognosis is not good.

Integration of weighted average: this method is an extension of the integration by average, with the main difference that the weighted average assigns importance weights to each of the ANFIS. These weights are assigned to a particular ANFIS based on several factors; the most important is the knowledge product experience. This integration method belongs to the well-known aggregation operators. However, there are some basic aspects of this approach, which are in need of better understanding. More specifically:

1) No standard methods exist for transforming human knowledge or experience into the fuzzy rule base and database of a fuzzy inference system.
2) There is a need for effective methods for tuning the membership functions (MF's) so as to minimize the output error measure or maximize a performance index.

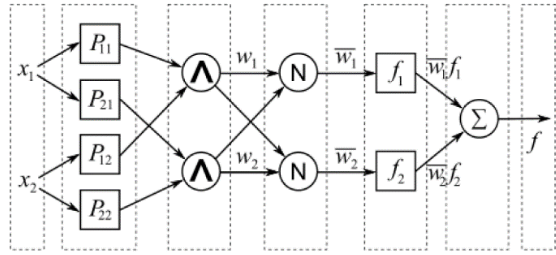

Fig. 5. Type-2 ANFIS Structure [19]

TABLE.1- Recently applied methods for uncertain time series forecasting.

Methodology	**Advantages**	**Limitations**
Ensembles of ANFIS [1]	New model for membership functions	No optimum parameter
Chaotic Time Series with ANFIS [2]	Reasonable Complexity	No data reduction approaches
GA-Type-1 and Type-2 in Ensembles of ANFIS [3]	Reliable outcomes for high order uncertainty	Complexity of model
PSO-IT2FNN with Fuzzy Integrators [4]	Combination of derivative free models and derivative based models	local optima and over-fitting problem
IT2FNN with Optimization of Fuzzy Integrators [5]	Reliable outcomes in uncertain condition	Doesn't model the benefit/cost along together in performance evaluation
Nearest Neighbor Network [6]	Using a neighbor network for obtaining more accurate results	High-order uncertainty is not modeled
Segmented Dynamic Time Warping (SDTW) [7]	A Slope-based PCA model	Limited to small data
Weighted dynamic time warping (WDTW) [8]	modified logistic weight function	Weights are crisp and not model well the associated uncertainty
Flexible Dynamic Time Warping (FDTW) [9]	Flexible model for various type of real-world problems	Limited to classification of time series
Fuzzy Clustering of Time Series [10]	shape-based clustering	Unsatisfactory results for long and mid-level time series.
Dynamic Time Warping Distances (DTWD) [11]	combination with one-nearest neighbor to handle the convincingly outperform	No reliable model for handling uncertainty
Fuzzy SVM-PSO Framework [12]	Using PSO for adjust the lengths of intervals	Time Complexity
Interval Type-2 Fuzzy ANN [13]	Reliable in chaotic time series prediction	No rule tuning
Type-2 Neuro Fuzzy-PSO (T2FNN-PSO) [15]	Unable to forecast the short-term time series	Reliable optimization model

4. Mathematical Model of Uncertain Fuzzy Time Series

Definition 1: Let be the universe of discourse, where $U = \{u1, u2, \cdot\cdot\cdot, un\}$, then a fuzzy set A of U is defined in formula (1):

$$A = \frac{fA(u1)}{u1} + \frac{fA(u2)}{u2} + \cdots + \frac{fA(un)}{un} \qquad (9)$$

where fA is the membership function of the fuzzy set A, $fA\colon U \rightarrow [0,1]$, $fA(ui)$ represents the degree of membership of ui belonging to the fuzzy set A, $fA(ui) \in [0, 1]$, $1 \leq i \leq n$.

Definition 2: Let $Y_{(t)}$ $(t = \ldots, 0, 1, 2, \ldots)$ be a subset of real number and be the universe of discourse in which fuzzy sets $f_{i_{(t)}}$ $(i = 1, 2, \ldots)$ are defined. Let F

(t) be a collection of (t) $(i = 1, 2, \ldots)$. Then, (t) is called a fuzzy time series on $Y(t)$ $(t = \ldots, 0, 1, 2, \ldots)$.

Definition 3 [2]: (t) is a fuzzy time series. Assume (t) is caused by $(t - 1), (t - 2), \cdots (t - n)$, then this fuzzy logical relationship can be given by

$$F(t) = \big(F(t-1) \times F(t-2) \times \cdots \times F(t-n)\big) \circ R(t, t-n) \qquad (10)$$

Where $\circ$ denotes the max-min composition operator, then (2) is called the n th order fuzzy time series forecasting model.

Definition 4: Let
$(t - 1) = A_{i_1}$ $(t - 2)$ $A_{i_2}, \ldots, (t - n)$
$A_{i_{in}}$ and (t) A_{i_0} , where $A_{i_1}, A_{i_2}, \cdots, A_{i_{in}}$ and A_i are fuzzy sets. The fuzzy logical relationship between $(t - 1), (t - 2), \cdots, (t - n)$ and (t) in definition 3 can be denoted by:

$$A_{i_1}, A_{i_2}, \cdots, A_{i_{in}} \rightarrow A_{i_0} \qquad (10)$$

where $A_{i_1}, A_{i_2}, \cdots, A_{i_{in}}$ and A_{i_0} are called the left-hand side (LHS) and the right-hand side (RHS) of the fuzzy logical relationship, respectively.

4.1 Type-2 Fuzzy Time Series

There is only one degree of membership value for a certain linguistic variable based on the concept of Type 1 fuzzy sets. While Type-2 fuzzy sets are adopted to convey the uncertainties in membership functions of Type 1 sets. According to the relations between Type 1 and Type 2 fuzzy sets, a Type 2 fuzzy time series forecasting model can be constructed.

Definition 5: A Type 2 fuzzy time series model can be seen as an extension of a Type 1 model. Relationships established by Type 1 observations are used by This Type 2 model. Type 2 forecasts are then calculated from these fuzzy relationships. Two operators are defined in definition 7 and 8 are used to include or screen out the FLRs in Type 2 model, shown as the followings. The lengths of intervals can directly influence the results of the forecasting. In order to improve the forecasting accuracy, some intelligent optimization methods such as GA and PSO, are applied to optimize the partition of universe in [6-8]. Based on this idea, in this paper, we use PSO [6] method to determine the subintervals. The detailed steps are shown as follows:

Define the universe of discourse U = [$Dmin$−$F1$, $Dmax$ +$F2$]. $Dmin$ and $Dmax$ are the minimum and maximum value of the training data, respectively. $F1$ and $F2$ are two positive real numbers. Assume that the number of intervals is n. k particles can be generated by the randomly generated initial position and velocity vectors. Calculate the fitness value for all the particles.

Definition 6: *N*-Order Fuzzy Relations. Let F(t) be a fuzzy time series. If F(t) is caused by F(t-1), Ft-2) ..., F(t-n), then this fuzzy relationship is represented by:

$$F(t-n), \ldots, F(t-2), F(t-1) - F(t) \qquad (11)$$

and is called an n-order fuzzy time series. The n-order concept was first introduced by Chen in [31]. N-order based FTS models are referred to as high order models.

Definition 7: Time-Invariant Fuzzy Time Series Suppose F(t) is caused by F(t-1) only and is denoted by F(t-1)-F(t), then there is a fuzzy relationship between F(t) and F(t-1) which is expressed as the equation:

$$F(t) = F(t-1)XR(t-1, t) \qquad (12)$$

The relation R is referred to as a first order model of F(t). If $R(t-1, t)$is independent of time t, that is, for different times t, and

$$t2, R(t1, , -1) = R(t2, t2-1) \qquad (13)$$

then F(t) is called a time-invariant fuzzy time series. Otherwise it is called a time-variant fuzzy time series.

Definition 8: Fuzzy Relationship Group (FLRG) Relationships with the same fuzzy set on the left-hand side can be further grouped into a relationship group. Relationship groups are also referred to as fuzzy logical relationship groups or FLRG 's in short. Suppose there are relationships such that:

$$A; \rightarrow Aj1, Aş\text{–}\ A2\ 4; \text{—}\ Ajn \qquad (14)$$

The same fuzzy set cannot appear more than once on the right-hand side or the relationship group. The term relationship group was first introduced by Chen in [3].

5. Experimental Results on Global Benchmark

As we know that the Mackey Glass system is infinite-dimensional (for the reason that it's a time-delay equation) therefore, has an infinite number of Lyapunov exponents or λ_i. All Mackey-Glass delay differential equation is chosen as the specific chaotic system that is used in the multi-step ahead prediction problem. The equation as the basic Mackey-Glass chaotic system [20] is given by:

$$\frac{dx(t)}{dt} = \frac{\beta x(t-\tau)}{1+x^n(t-\tau)} - \gamma x \tag{15}$$

where the constants β, γ, n, and τ, determined the behavior characteristics of the chaotic system. This equation is very sensitive to initial condition of the constant parameters, and in this work, these parameters are taken β=0.2, γ=0.1, n=10, and τ=17, respectively. Figure 1 shows the behavior of state trajectory in Mackey-Glass delay differential equation. The test of different fuzzy systems we applied a simulation of times-series data using the following form of the MG nonlinear delay differential equation:

$$x(t) = \frac{0.2x(t-\tau)}{1+x^{10}(t-\tau)} - 0.1x(t) \tag{16}$$

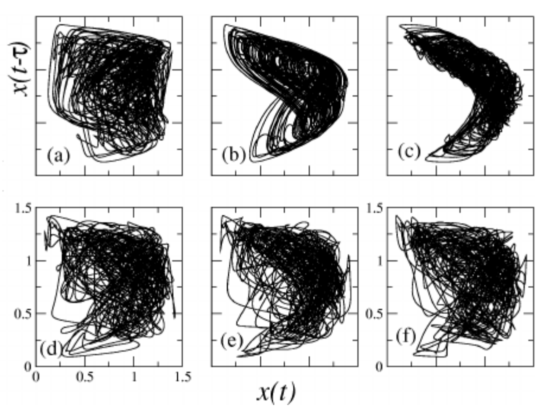

Fig.7. Chaotic Mackey-Glass delay differential equation with diverse noise rate

Simulate the time-series for 1200 samples using the following configuration:
Sample time $t_s = 1$ sec, Initial condition $x_0 = 1.2$ and $\tau = 20$, where $x(t-\tau) = 0$ for $t < \tau$.

$$\text{MAE} = \frac{1}{2}\sum_{t=1}^{n} |(A_t - P_t)| \tag{17}$$

$$\text{MSE} = \frac{1}{2}\sum_{t=1}^{n} (A_t - P_t)^2 \tag{18}$$

$$\text{RMSE} = \sqrt{\tfrac{1}{2}\textstyle\sum_{t=1}^{n} (A_t - P_t)^2} \tag{19}$$

$$\text{MPE} = \frac{100\%}{n}\sum_{t=1}^{n} \frac{(A_t - P_t)}{A_t} \tag{20}$$

$$\text{MAPE} = \frac{100\%}{n}\sum_{t=1}^{n} \mathrm{I}\,\frac{(A_t - P_t)}{A_t} \tag{21}$$

where *A* is the desired prediction, the forecast of the fuzzy inference is *P*, the time variable is *t*, and the size of the time series is *n*.

TABLE-2 Computational results of different fuzzy methods applied on Mackey glass benchmark

METRICS	Type-1 Fuzzy	Type-2 Fuzzy	Neuro-Fuzzy	Fuzzy-GA	Type-2 ANFIS
RMSE	0.0157	0..0132	0.0121	0.0130	0.0107
MSE	0.0218	0.0184	0.0190	0.0177	0.0104
MAE	0.0147	0.0120	0.0128	0.0030	0.0020
MPE	0.0151	0.0144	0.0124	0.0098	0.0136
MAPE	1.5981	0.2369	0.2467	0.2202	0.1202

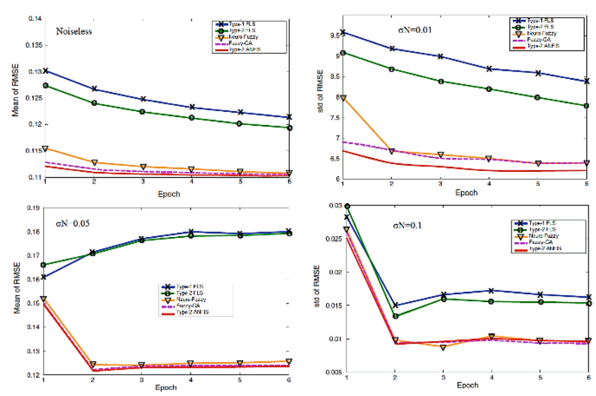

Fig. 8. Performance evaluation of different Fuzzy and hybrid fuzzy systems applied on chaotic time-series

5.1 Critical analysis

A primary objective of the time-series modeling approach is to build a classifier that can provide earliness while maintaining a desired level of reliability or accuracy. However, some approaches [10], [11], [12], [13] do not ensure the reliability, but they are capable enough to classify an incomplete time series. Recently, the researchers in approaches [14], [15], [16], [17] have successfully employed deep learning techniques for the time-series modeling and classification. These approaches have unfolded a new direction for further research in this area. Analysis of the miscellaneous time-series classification approaches is presented in Table 1.

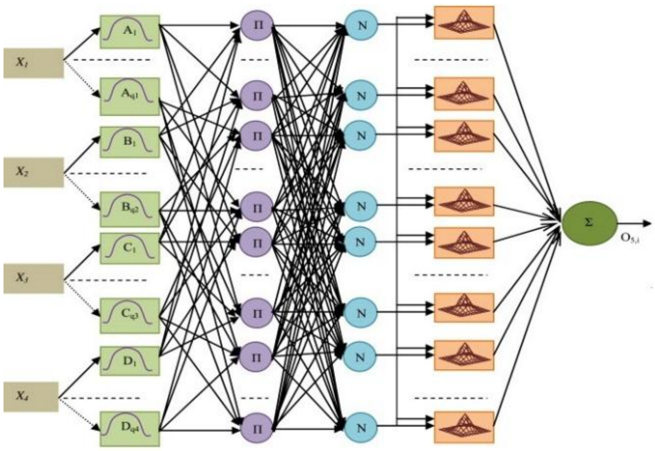

Fig. 9 A Fuzzy Deep-Learning Approach

6. Conclusion

This research is organized into three major categories depending upon whether type-1 fuzzy time series, type-2 fuzzy time-series and hybrid fuzzy time-series. we discussed many major time-series modeling in literature so far; they are: pattern discovery (clustering), classification, rule discovery and summarization. They are fuzzy models on multi-attribute time series, mining on time series data stream and short and long-term predictions. The experience gained in working with the ensemble hybrid systems indicates that this approach is highly efficient due to its high representation capacity, and it has a structure that makes it highly parallelizable in today's platforms.

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